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Image Quality Estimation by Entropy and Redundancy Calculation for Various Wavelet Families

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Abstract: **In the present work we analyze the performance of orthogonal and biorthogonal wavelet filters for image compression on variety of test images. The test images are of different size and resolution. Three distortion measures are used: entropy of reconstructed image, energy retained and redundancy. Analysis is done on the basis of entropy and redundancy calculation and it is found that Biorthogonal wavelets are superior as compare to orthogonal ones.**

Key Words: Wavelet, Orthogonal, Biorthogonal, Entropy, Redundancy, Energy Retained.

I. Introduction

The main issue for the development of good image compression system is to compress the image while keeping the total information preserved. For lossless compression the compression scheme should be reversible as well as probabilistic in nature to reduce the entropy, preserve energy and exact recovery of image after decompression [1][2]. Efficient representation of images depends on total energy distribution of different image components hence with compression performance. The compression performance can be measured in terms of energy retained, entropy and redundancy. The transform used for image compression and decompression should conserve the total energy during transform-inverse transform operation and are reversible in nature. Any suitable image transform redistributes the energy of image or data into transform coefficients of which some coefficients are having most of the energy and rest having very less and zero energy. Thus in transform matrix there are very few values to be encoded hence achieving compression. As the correlation in data set is high, it is possible to have more number of zeros in transformed matrix. Any transform based image compression system will show better performance with compression if transform coefficients are having more and more number of zeros and maximum retained energy[3].

In compression the fundamental idea is to reduce the redundancy and irrelevancy. Removing the duplication from the image is called as redundancy removal. Omitting the part of the signal that will not be noticed by the human visual system (HVS) is called irrelevancy reduction [4]. In still image, compression is achieved by removing spatial and spectral redundancy. Also, the entropy of image plays an important role in image compression, as it decides the maximum codeword length [5]. If image entropy is small, less number of bits are required to represent the image, means more compression can be performed. For entropy estimation the two fundamental concepts can be used. These are Shannon's Entropy and Relative Entropy. The first one aims to measure the information content of an image histogram and the latter enables us to describe the discrepancy between the image histograms of different images or in the same image [6].

II. Discrete Wavelet Transform

Wavelet transform of still images is based on multiresolution analysis and splitting the image signal in low and high frequency components through convolution of the signal with dilated filter. The distinction between discrete cosine transform (DCT) used for JPEG and wavelet transform is in shape of function that is used for transformation. The Discrete Wavelet Transform (DWT) of image signals produces a non-redundant image representation, which provides better spatial and spectral localization of image formation, compared with other multi scale representations such as Gaussian and Laplacian pyramid [7]. Recently, Discrete Wavelet Transform has attracted more and more interest in image de-noising [8]. The DWT can be interpreted as signal decomposition in a set of independent, spatially oriented frequency channels. The signal is passed through two complementary filters and emerges as two signals, approximation and Details. This is called decomposition or analysis. The components can be assembled back into the original signal without loss of information. This process is called reconstruction or synthesis. The mathematical manipulation, which implies analysis and synthesis, is called discrete wavelet transform and inverse discrete wavelet transform.

III. Image Compression Methodology

Wavelet transform is a pair of filters. The way we compute the wavelet transform by recursively averaging and differentiating coefficients is called the filter bank [9], where one is a low pass filter (lpf) and the other is a high pass filter (hpf). Each of the filters is down sampled by two. Each of those two output signals can be further transformed. Similarly, this process can be repeated recursively several times, resulting in a tree structure called the decomposition tree. Wavelet transform can be used to analyze or decompose signals and images called decomposition [10] [11][12].The same components can be assembled back into the original signal without loss of information; this is called reconstruction or synthesis. The image compression methodology involves mainly thress steps [13]: transformation, thresholding and encoding as shown in figure 1.

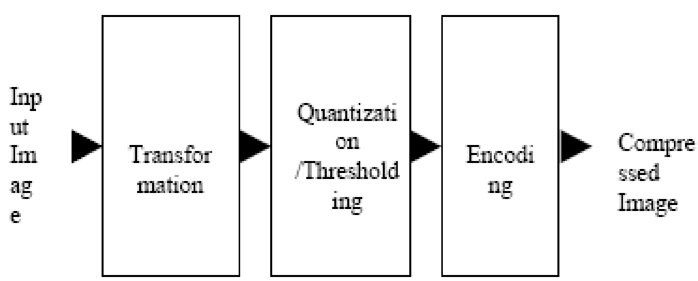

Fig 1: Representation of image compression methodology

A. Transformation:

The structure of Wavelet can be represented as a four channel perfect reconstruction of filter bank. Each filter is 2D with subscript indicating the type of filter (HPF or LPF) for separation of horizontal and vertical components. The resulting four-transform components consist of all possible combinations of high and low pass filtering in the two directions. By using these filters in one stage an image can be decomposed into four bands. There are three types of details of images for each resolution Diagonal (HH), Vertical (LH) and Horizontal (HL). The operations can be repeated on the low low (LL) i.e. on approximation band using the second identical filter bank [14].

The decomposition process can be iterated, with successive approximations being decomposed. However, in practice, more than one decomposition level is performed on the image data. Successive iterations are performed on the low pass coefficients (approximation) from the previous stage to further reduce the number of low pass coefficients. Since the low pass coefficients contain most of the original signal energy, this iteration process yields better energy compaction. The quality of compressed image depends on the number of decompositions [15]. Compression of an image can be obtained by ignoring all coefficients less than the threshold value. If we use decomposition iteration, it will be more successful in resolving DWT coefficient because Human Visual System (HVS) is less sensitive to removal of smaller details. Decomposition iterations depend on the filter order. Higher order does not imply better image quality because of the length of the wavelet filter. This becomes a limiting factor for decomposition. Usually, five levels of decompositions are used in current wavelet based image compression [16] [17] [18].

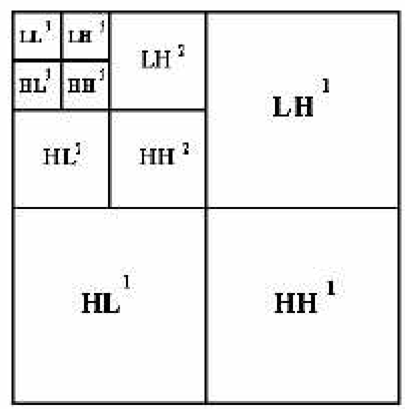

Figure 2: 2D-DWT with 3-Level decomposition

B. Thresholding

In certain signals, many of the wavelet coefficients are close or equal to zero. Through a method called thresholding, these coefficients may be modified so that sequence of wavelet coefficients contains long strings of zeros. Through a type of compression known as entropy coding, these long strings may be stored and sent electronically in much less space.

In image compression technique two types of thresholding are used as:

- Hard Thresholding
- Soft Thresholding

In hard thresholding technique, a tolerance is selected. Any wavelet whose absolute value falls below the tolerance is set to zero with the goal to introduce many zeros without losing a great amount of detail. There is not a straightforward easy way to choose the threshold, although the larger the threshold that is chosen the more error that is introduced into the process.

Another type of thresholding is soft thresholding. Once again a tolerance h is selected. If the absolute value of any entry is less than the tolerance, than that entry is set to zero. All other entries d, are replaced with sign d-h. Soft thresholding can be thought of as a translation of the signal towards zero by the amount h.

C. Encoding

Wavelets and thresholding help process the signal, but up until this point, no compression has yet occurred. One method to compress the data is Huffman entropy coding [19][20]. With this method, and integer sequence, q, is changed into a shorter sequence, e, with the numbers in e being 8 bit integers. The conversion is made by an entropy coding table. Strings of zeros are coded by the numbers 1 through 100, 105, 106, while the non-zero integers in q are coded by 101 through 104 and 107 through 254. In Huffman entropy coding, the idea is to use two or three numbers for coding, with the first being a signal that a large number or long zero sequence is coding. Entropy coding is designed so that the numbers that are expected to appear the most often in q, need the least amount of space in e.

IV. Wavelet Families

The selection of wavelet function is crucial for performance in image compression [21]. There are a number of basis that decides the selection of wavelet for image compression. Since the wavelet produces all wavelet functions used in the transformation through translation and scaling, it determines the characteristics of the resulting wavelet transform. Therefore, the details of the particular application should be taken into account and the appropriate wavelet should be selected in order to use the wavelet transform effectively for image compression. Important properties of wavelet functions in image compression applications are compact support, symmetry, orthogonality, regularity and degree of smoothness [22][23] [24].

In our experiment four wavelet families are examined: Daubechies Wavelet (DB), Biorthogonal Wavelet (BIOR), Coiflet Wavelet (COIF) and Symlet (SYM). The DB, BIOR, and COIF wavelets are families of orthogonal wavelets that are compactly supported. These wavelets are capable of perfect reconstruction. DB is asymmetrical while Coiflet is almost symmetrical. Scaling and wavelet functions for decompositions and reconstruction in the BIOR family can be similar or dissimilar. Daubechies wavelets are the most popular wavelets and represent the foundation of wavelet signal processing and are used in numerous applications. The wavelets are the selected based on their shape and their ability to compress the image in a particular application. Figure 3 illustrates some of the commonly used wavelet functions used in our experiments.

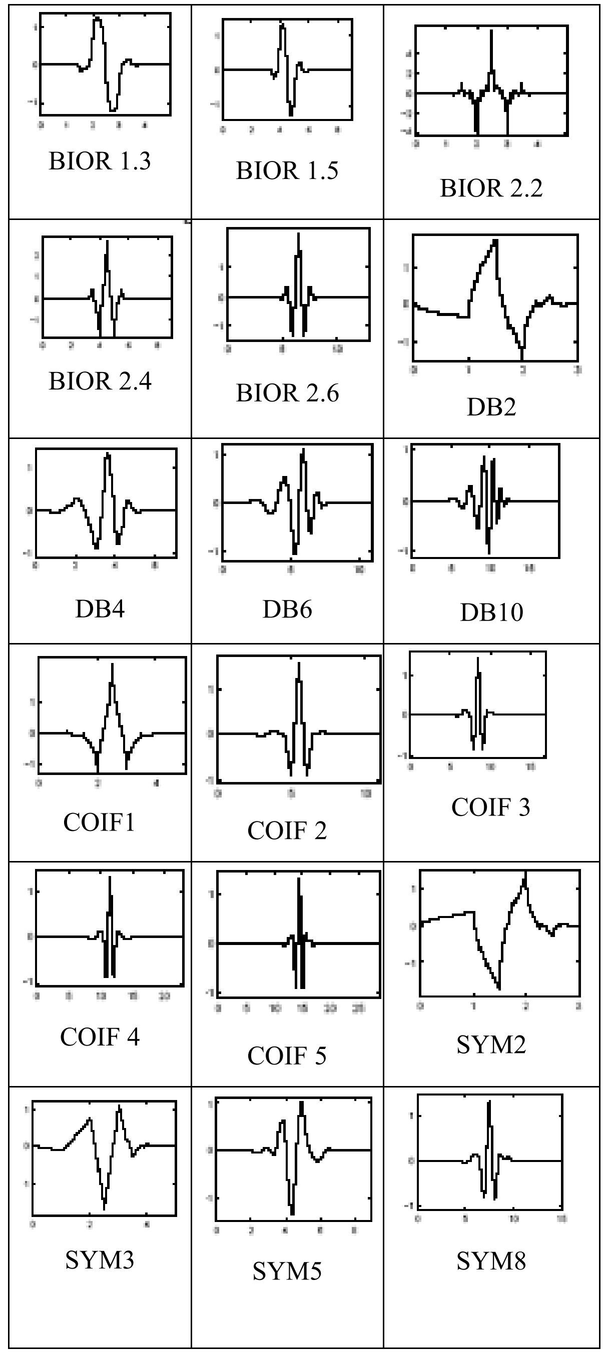

Figure 3: Wavelets families used in our experiments

V. Wavelet Properties

To achieve a high compression rate, it is often necessary to choose the best wavelet filter bank and decomposition level, which will play a crucial role in compressing the images [22]. The selection of wavelet filters plays a crucial part in achieving an effective coding performance, because there is no filter that performs the best for all images. The choice of optimal wavelets has several criteria. The main criteria are:

(1) Orthonormality
(2) Filter Length
(3) Vanishing order or moment
(4) Smoothness
(5) Decomposition level
(6) Regularity

Wavelet Filter can be used to analyze or decompose signals and images called decomposition. The same components can be assembled back into the original signal without loss of information, which is called reconstruction or synthesis. Shorter synthesis basis functions are desirable for minimizing distortion that affects the subjective quality of the image. Longer filters are responsible for ringing noise in the reconstructed image at low bit rates. Each wavelet family is parameterized by an integer N called the filter order, which is proportional to the length of the filter. The length of the filter is related to the degree of the smoothness of the wavelet and can affect the coding performance. This relation is different for different wavelet families and non-smoothness basis function introduces artificial discontinuities are reflected as spurious artifacts in the reconstructed images. Higher filter order gives more energy and increases the complexity of calculating the DWT coefficients, while lower order preserves the energy. i.e. it preserves the important edge information. Therefore, we must take care of wavelets in image compression application concerning that good balance between filter orders, degree of smoothness and computational complexity. These properties depend on the image contents. Vanishing order is the measure of compaction property of the wavelet and it corresponds to the number of zeros in the LL sub band.

Filter Response is another critical property that affects the subjective quality of the reconstructed image. The filter responses approach the ideal rectangular response with the increase in the number of zeros and these numbers of zeros also corresponds to vanishing order of the wavelet. Wavelet Transform can be used to analyze or decompose signal and image called decomposition. The same components can be assembled back into the original signal without loss of information called reconstruction or synthesis. The decomposition process can be iterated with successive approximations being decomposed. However, in practice more than one decomposition level is performed on the image. Successive iterations are performed on approximation coefficients; this successive iterations process yields better energy compaction. The quality of the compressed image depends on the number of decompositions, and these decomposition iterations depend on the filter order. Higher order does not imply better image quality because of the length of wavelet filter. This becomes a limiting factor for decomposition. Usually five levels of decomposition are used in current wavelet based compression. Regularity is one of the properties of the wavelet; greater regularity often does not improve the visual quality.

VI. Distortion Characterizations

A. Entropy

The image entropy can be estimated as [25]:

$$H(X) = -\sum_{i=0}^{255} p_i \log p_i \qquad (1)$$

$$p_i = \frac{N_i}{N} \qquad (2)$$

where the number of pixels with grey level is N_i ; the total number of pixels in the image is N; p_i is the probability of occurrence of one gray level intensity , and

$$\sum_{i=0}^{255} p_i = 1, \qquad 0 \le p_i \le 1 \qquad (3)$$

The entropy of a given source is affected by the number of elements in X. Thus a normalized measure, redundancy, is better for comparing multiple sources.

B. Energy Retained

When compressing with orthogonal wavelets the energy retained is [26] :

$$\frac{100*(\mathit{vector\text{-}norm}(\mathit{coeff\_of\_the\_current\_decomposition}))^2}{(\mathit{vector\text{-}norm}(\mathit{original\_signal}))^2} \qquad (4)$$

C. Redundancy

The redundancies in an image can be identified as spatial redundancy, spectral redundancy and temporal redundancy. Image compression research aims at reducing the number of bits needed to represent an image by removing the spatial and spectral redundancies as much as possible. Since the focus is only on still natural image compression, the temporal redundancy is not considered as it is used in motion picture [27]. Information redundancy, r, is

$$r = b - He \qquad (5)$$

where b is the smallest number of bits for which the image quantization levels can be represented.

VII. Entropy and Histogram

In an image processing context, the histogram of an image normally refers to a histogram of the pixel intensity values. This histogram is a graph showing the number of pixels in an image at each different intensity value found in that image. For an 8-bit grayscale image there are 256 different possible intensities, and so the histogram will graphically display 256 numbers showing the distribution of pixels amongst those grayscale values. Histograms can also be taken of color images --- either individual histogram of red, green and blue channels can be taken, or a 3-D histogram can be produced, with the three axes representing the red, blue and green channels, and brightness at each point representing the pixel count. The exact output from the operation depends upon the implementation. It may simply be a picture of the required histogram in a suitable image format, or it may be a data file of some sort representing the histogram statistics.

The horizontal axis of the graph represents the tonal variations, while the vertical axis represents the number of pixels in that particular tone. The left side of the horizontal axis represents the black and dark areas, the middle represents medium grey and the right hand side represents light and pure white areas. The vertical axis represents the size of the area that is captured in each one of these zones. Thus, the histogram for a very bright image with few dark areas and/or shadows will have most of its data points on the right side and center of the graph. Conversely, the histogram for a very dark image will have the majority of its data points on the left side and center of the graph.

From Equation (1), entropy of an image is measured using 256 bins, which correspond to the 256 quantize levels, and the count of each level is divided by *N* to give the probability p_i . The maximum entropy value is achieved when the gray level values are distributed uniformly, and the minimum entropy value is achieved when the image consists of only one gray level value and the histogram shows one bin. Therefore, the entropy of a gray scale image changes with the distribution of the histogram of the image. When an image is natural image, the histogram typically consists of combinational Gaussian distributions in the range (0, 255). In image compression, low image entropy suggests that a high compression ratio can be achieved using efficient coding. The image with lower entropy implies lower image quality for lower information content. Here the histograms of test images Lena, Cameraman, Pepper and Wbarb are shown. The normalized histogram of Lena has more rise and fall; it has a more uniform distribution with more consistent numbers in each bin. Thus, the entropy of Lena image is higher than the entropy of other images. So, more compression can be found in Lena image and it can be concluded that the compression performance depend over image content.


Figure 4: Lena Image

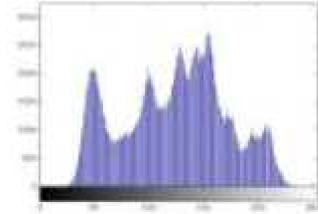
Figure 5: Histogram of lena image


Figure 6: Cameraman Image


Figure 7: Histogram of cameraman image


Figure 8: Barbara image

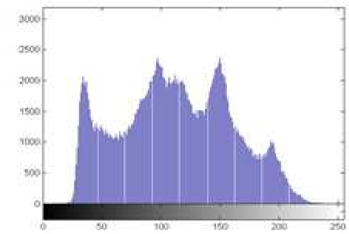
Figure 9: Histogram of Barbara image

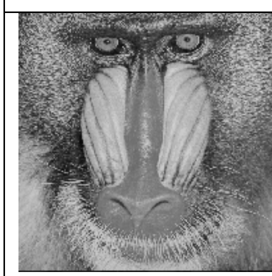
Figure 10: Mandrill image

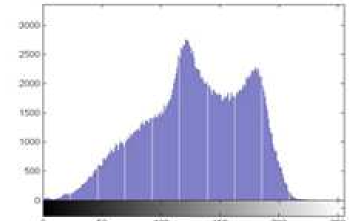
Figure 11: Histogram of Mandrill iImage

VIII. Materials and Methods

The following tasks are identified to accomplish efficient image compression.

1. Load the image from the user.
2. Apply 2D DWT over the whole image.
3. Set the threshold value 10,20,30,40,50,60,70,80,90 and 100.
4. This simulates the process of compressing by factors of 1/10, 1/20, 1/30, 1/40, 1/50, 1/60, 1/70, 1/80 1/90 and 1/100 respectively.
5. Display the resulting images and comment on the quality of the images.

6. Calculate ER and redundancy values with varying threshold values for corresponding reconstructed images.
7. Entropy of reconstructed image is estimated from the gray level histogram.
8. Repeat the same process for various images with various filter orders and compare its performance.

IX. Experimental Results

The proposed algorithm is used to measure objective quality of compressed images. The quality parameters are energy retained, entropy and redundancy. Importance of these parameters has been stated earlier. The algorithm is applied on test images using wavelet toolbox. However, experimental results for Lena image are shown here.

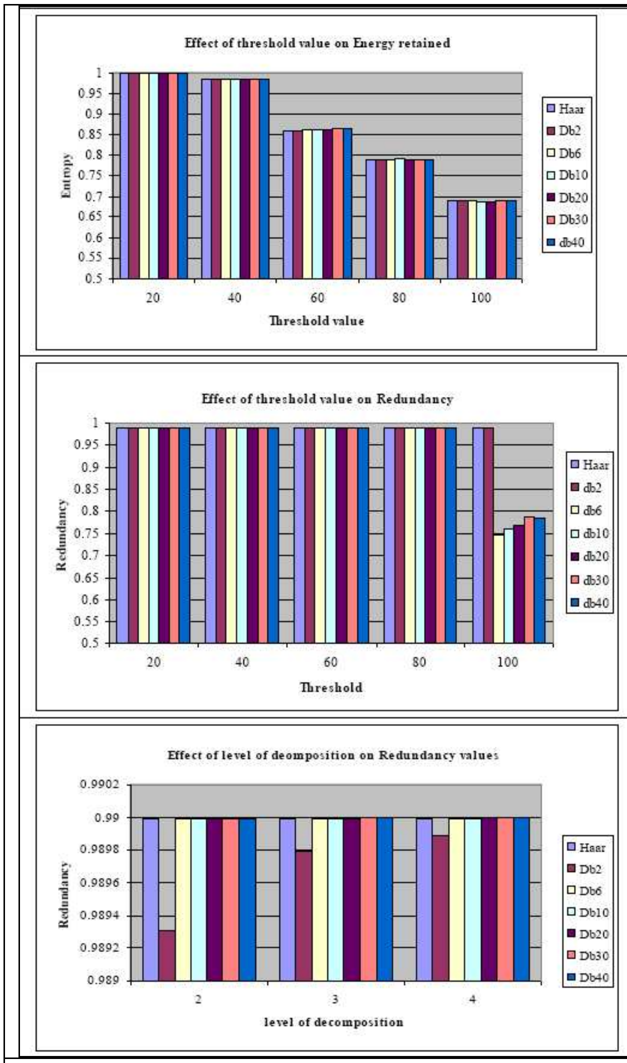

Figure 12: Experimental results for Daubechies wavelet family

X. Results and Discussions

Here experiments are performed on standard test images, Barbara, Mandrill, Lena and Pepper. Results indicate that the average entropy is the minimum in case of compression by Biorthogonal family. The entropy is decreasing with increasing threshold but decrement in biorthogonal wavelets is fast as compare to orthogonal wavelets. Also, the effect of level of decomposition over entropy is shown and it is fond that the reduction in entropy value with increased level of decomposition is fast in biorthogonal wavelets.

Again, the Haar wavelet transform is the simplest one to implement, and it is the fastest. However, because of its discontinuity, it is not optimal to simulate a continuous signal. Based on our experiments, Haar wavelet has obtained the worst compression result, which proves the above statement. Also, it is found that visual quality of compressed image deteriorates using longer filters e.g. visual quality of compressed image is superior for Db2 filter as compare to Db40, although it is having higher entropy values.

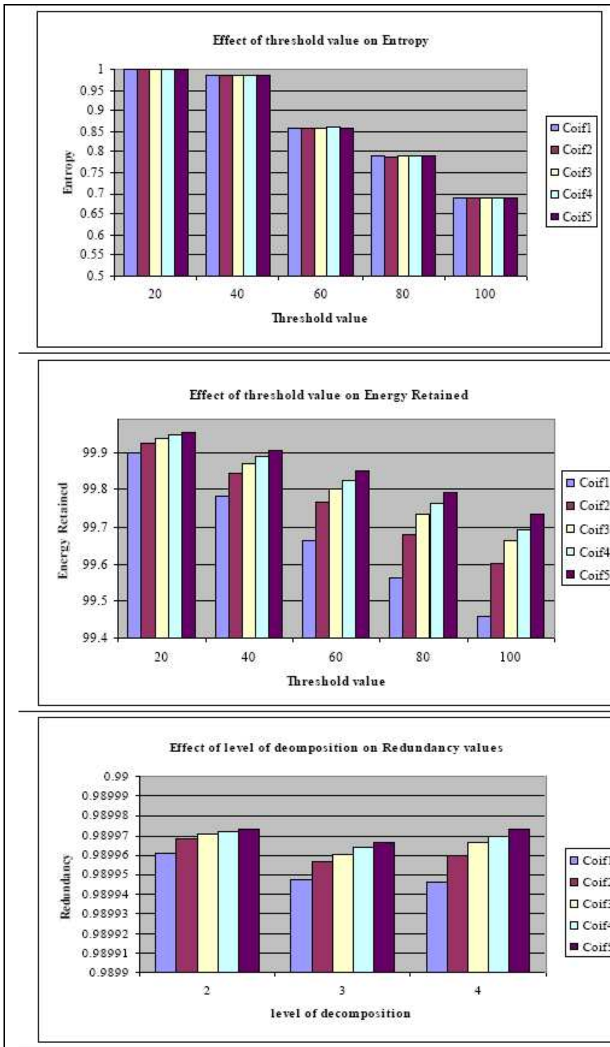

Figure 13: Experimental results for Coiflet wavelet family

Also, here the probability of occurrence of events is used to calculate the coding redundancy. For reducing the redundancy the 2D pixel array that is normally used for human viewing and interpretation must be transformed into a more efficient format. The visual redundancies can be reduced using the fact that the human eye does not respond with equal sensitivity to all visual information. Using this fact redundancy calculations have been made and coding redundancy is found minimum for Biorthogonal 2.6 wavelet. Reason behind this performance is that Biorthogonal wavelets can use filters with similar or dissimilar order for decomposition (Nd) and reconstruction (Nr). Therefore Biorthogonal wavelet is parameterized by two numbers and filter length is {max (2Nd,2Nr) +2}. Higher filter orders give higher degree of smoothness. Wavelet based image compression prefers smooth functions of relatively short support and so the Biorthogonal wavelets perform better.

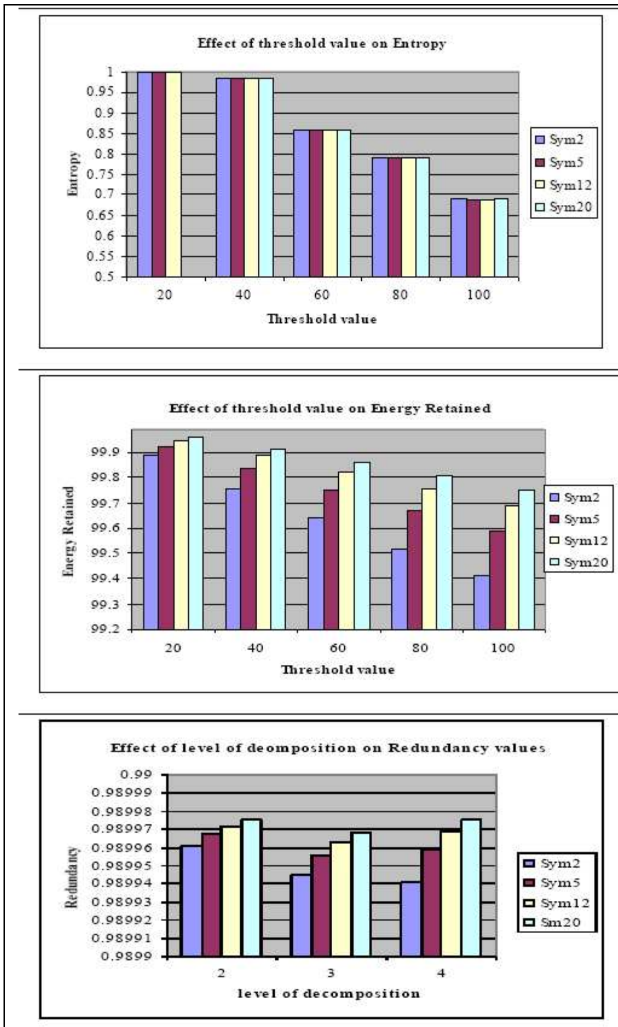

Figure 14: Experimental results for Symlet wavelet family

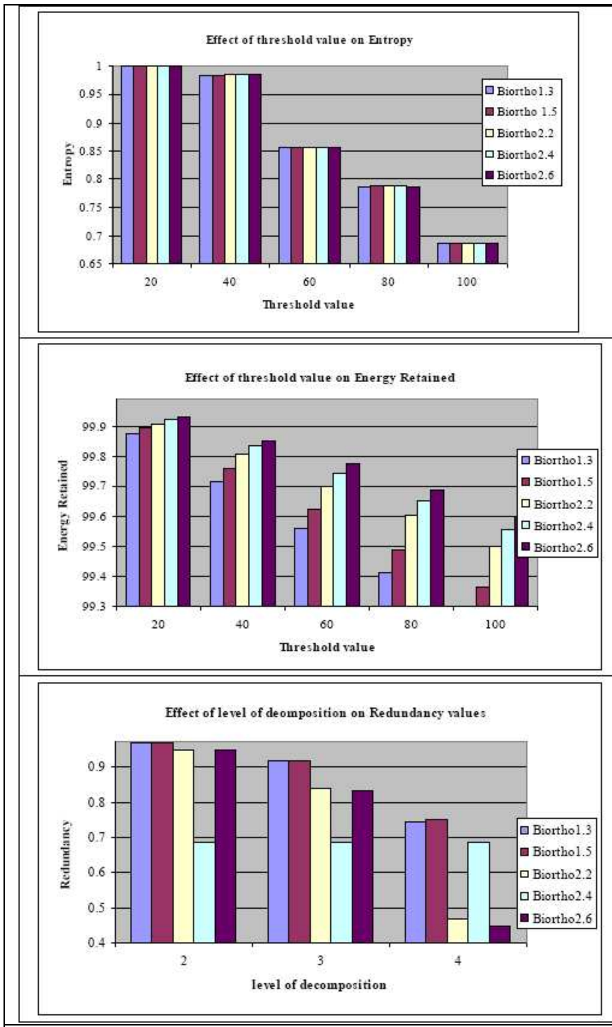

Figure 15: Experimental results for Biorthogonal wavelet family

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