Design of Fixed Order Controller by Using Particle Swarm Optimization and System Simulation

Faizullah Mahar1, Syed Saad Azhar Ali2, Ayaz Hussain3

1 Department of Electronic Engineering, Iqra University, Shaheed-e-Millat Road, Defence View Karachi 75500, Pakistan
ch_it2001@yahoo.com

2 Department of Electronic Engineering, Iqra University, Shaheed-e-Millat Road, Defence View Karachi 75500, Pakistan
saadazhar@iqra.edu.pk

3 Department of Electrical Engineering, Balochistan University of Engg. & Tech. Khuzdar, Pakistan
engr_ayaz@yahoo.com

Abstract: Design of fixed order controllers is an open problem which takes controller complexity into account. Different solutions exist in literature for the fixed order controller design problem. However, major difficulty in these results is that they are not simple to develop an algorithm that guarantees the decisive solutions coupled with computations. To overcome this difficulty $H_\infty$ loop shaping control under an order specified controller has been proposed. The performance specification is formulated as cost function, which is to be minimized Design. Particle swarm optimization has been adopted for optimization of cost function and parameters of the specified controller. To check the success of proposed techniques, resulting controller parameters are evaluated via system simulations. Simulation results demonstrate the efficiency of proposed technique.

Keywords: Particle swarm, cost function, velocity, inertia and controller.

I. Introduction

There is close link between the history of the development of control theory and computational methods available at that time [1]. Designing controller for stabilizing complex plant and achieving a specific performance is generally a difficult task. Both computation and implementation of such controllers are serious issues to be dealt with control system design. There are different approaches to simplify the design process; an alternative is to minimize the closed loop performance index by a fixed order controller [2].

The controllers obtained from standard design techniques have generally much higher order than that of the plant. As order of plant may be high, designing full order controller narrows the possibility of use in practical application, that's why there has been increasing and considerable interest in designing low, fixed order controllers but there are basic difficulties inherent to low, fixed order controller design [3].

In the field of control engineering fixed order controller design are gaining increasing interest, one of the reasons is that classical controller design techniques lead to higher order controllers and are complicated to put into operation [4, 5].

Designing controller for stabilizing complex plant and achieving a specific performance is generally a difficult task. There are different approaches to simplify the design process. An alternative is to minimize the closed loop performance index by a fixed order controller [6, 7]. However, there are basic difficulties intrinsic to low; fixed order controller design, such as to find the best possible values of controller gain or optimal performance.

Designing a controller means choosing the suitable gains. The main thing to note is that if the calculated value of gain is too large, the response will vary with high frequency. On the other hand, having too small gains would mean longer settling time. Thus, finding the best possible value for gain is the most important concern in controller design [8]. Generally, the overall design procedure is iterative between controller design and cost function (CF) evaluation [9]. If performance is not satisfactory one has to fine-tune the controller parameters after using Ziegler-Nichols (Z-N) tuning rule, which gives an educated guess for controller parameter values or with adjusting some weighting functions in CF used to synthesis the controller [10].

A population of particles is initialized with random positions and velocities and a function is evaluated, using the particle’s positional coordinates as input values. Positions and velocities are adjusted and the function evaluated with the new coordinates at each time step. When a particle discovers a pattern that is better than any it has found previously, it stores the coordinates in a vector. The difference between (the best point found by so far) and the individual’s current position is stochastically added to the current velocity, causing the trajectory to oscillate around that point. Further, each particle is defined within the context of a topological neighborhood comprising itself and some other particles in the population.
A basic feedback SISO system shown in Fig. 1, the plant has transfer function \( P \) and the controller has transfer function \( K \). This is algebraically identical to the standard two-term form.

When the particle swarm operates on an optimization problem, the value of \( \bar{P} \) is constantly updated, as the system evolves toward an optimum. In order to further simplify the system and make it understandable, we set \( \bar{P} \) to a constant value in the following analysis. The system will also be more understandable if we make \( \phi \) a constant as well; where normally it is defined as a random number between zero and a constant upper limit, we will remove the stochastic component initially and reintroduce it in later sections. The effect of \( \phi \) on the system is very important and much of the present paper is involved in analyzing its effect on the trajectory of a particle. The system can be simplified even further by considering a one-dimensional (1-D) problem space and again further by reducing the population to one particle. Thus, we will begin by looking at a stripped-down particle by itself, e.g., a population of one 1-D deterministic particle, with a constant \( \phi \). Thus, we begin by considering the reduced system

\[
\begin{align*}
\dot{v}(t+1) &= v(t) + \phi(p - x(t)) \\
\dot{x}(t+1) &= x(t) + v(t+1)
\end{align*}
\]

Where \( P \) and \( \phi \) are constants. No vector notation is necessary and there is no randomness [14].

In [14], found that the simplified particle’s trajectory is dependent on the value of the control parameter \( \phi \) and recognized that randomness was responsible for the explosion of the system.

In recent years, particle swarm optimization (PSO) procedure appeared as promising technique for managing the optimization problems. Recently PSO is used to design robust controllers for dynamics of multi-machine system. PSO is population based optimization technique that has many advantages over other classical optimization procedures. The PSO is a general purpose optimizer that solves the wide range of optimization problems, thus the PSO technique can be easily adapted to suit various categories of optimization problem [15, 30].

Most industrial applications use simple controllers such as PI and PID controllers. Other complicated controllers e.g. \( H_\infty \) etc are hardly used. The goal in this paper is to design a controller with simple structure which has low, fixed order and retains stability. The fixed order \( H_\infty \) controller design approach is proposed by using PSO along with the concepts developed in [16, 32].

The nominal plant has been shaped by choosing the weighting functions; cost function to be minimized and maximized as the stability margin of the shaped plant. A set of controller parameters \( p \) in pre-specified controller \( K \) is optimized by using PSO. The designed controller has been implemented in the nominal plant.

\[
G(s) = \frac{551 - 13e^{-0.125}}{s^3 + 43.26s + 536.9}
\]

A basic feedback SISO system shown in Fig. 1, the plant has transfer function \( P \) and the controller has transfer function \( K \). The signal \( v \) represents a disturbance acting on the system and \( z \) is control system output.
The objective of this paper is to design a robust controller, which holds simple structure with lower order that maintains the robustness and meet the performance specifications. Our scope of design is a single output single input (SISO) plant, but it can be extended to a multi input multi output (MIMO) plant.

Figure 1. SISO feedback loop

The problem is formulated as the minimization of performance index subject to $H_\infty$ robust stability constraints. Hence the problem of the synthesis of controller is one how to solve above minimization problem. In this paper evolutionary techniques are used because these techniques are robust and likely to converge at global optimum Particle Swarm Optimization based controllers, thereby providing improved performance with respect to overshoot, settling time and oscillations.

The paper is arranged as follows: following the introduction about background literature and problem statement is presented in last paragraph. Section II describes $H_\infty$ robust control problem, the conventional $H_\infty$ loop shaping design procedure is presented in section III, section VI discusses the detailed procedure of proposed schemes to design of low, fixed order controller using Particle swarm optimization, section V presents discussion on simulation results, finally conclusion is summarized in section VI.

II. Standard $H_\infty$ robust control problem

Considering a generalized plant $P(s)$ in the general $H_\infty$ configuration as shown in Fig. 2, it will include plant model and disturbance model and the interconnection structure between the plant and the controller. Moreover, if $P(s)$ is being used to formulate a design problem, then it will also include weighting functions. $w$ denotes exogenous inputs; commands, disturbances and noise, and $z$ denotes exogenous outputs error signals to be minimized and $y$ is controller inputs for the general configuration, for example commands, measured plant outputs, measured disturbances etc.[17]. The objective is to find the stabilizing controller to minimize the output $z$; it is equivalent to minimizing the $H_\infty$ norm of the transfer function from $w$ to $z$ [18].

The closed loop transfer function from $w$ to $z$ can be obtained directly as:

$$Z = F_I(P, K)w$$

(5)

Where $F_I(P, K) = P_{11} + P_{12} K (I - P_{22} K)^{-1} P_{21}$ is called the lower fractional transformation of $P$ and $K$. Therefore, the optimal $H_\infty$ control problem is to minimize the $H_\infty$ norm of $F_I(P, K)$, i.e., $\|F_I(P, K)\|_\infty$

A. $H_\infty$ Robust Stabilization

In this approach the shaped plant is formulated as a normalized co -prime factor which separates the plant $G_2$ into normalized factors nominator $N$ and denominator $M$. One of the important uncertainties is co-prime factor uncertainty or numerator denominator perturbations [19, 31]; this uncertainty is repeatedly used in controller design procedure, where the objective is to maximize the magnitude of uncertainty such that robust stability is maintained [20].

The normalized co-prime factor robust stabilization problem

The normalized co-prime factorization of the shaped plant is $G_2 = \frac{N}{M}$, then a perturbed plant $G_2$ is then can be written as:

$$G_\Delta = (N + \Delta N)(M - \Delta M)^{-1}$$

(6)

Where, $\Delta N$ and $\Delta M$ are stable unknown transfer functions representing the uncertainty in the nominal plant model $G_2$, satisfying $\|\Delta N\|_\infty,\|\Delta M\|_\infty < \varepsilon$ where $\varepsilon$ is uncertainty boundary called stability margin. To maximize this class of perturbed model such that the configuration shown in Fig. 3 is stable, a controller $K_{\Delta}$ that stabilize the nominal closed loop system and minimizes gamma must be found by Eq. (7),

$$\gamma = \|\begin{bmatrix} I & (I + G_\Delta K_{\Delta})^{-1} M^{-1} \end{bmatrix} \|_{H_\infty}$$

(7)

Where $\gamma$ is the $H_\infty$ norm from $\Delta$ to $(\Delta N, \Delta M)$ and $(I + G_2 K_{\Delta})^{-1}$ is the sensitivity function, the lowest achievable value of gamma and corresponding maximum stability margin epsilon are calculated by Eq. (8)

$$\gamma = \varepsilon_{\max} = \sqrt{1 + \rho(XZ)}$$

(8)
The infinity norm of transfer function negligible, then

\[ G(s) = D + C(SI - A)^{-1}B \]  

(9)

\[ Z \] \text{is the unique positive definite solution to the following algebraic Riccati equation [21]}

\[ (A - BS^{-1}D^T C)Z + Z(A - BS^{-1}D^T C)^T - ZC^TR^{-1}CZ + BS^{-1}B^T = 0 \]  

(10)

\[ X \] \text{is the unique solution of the following algebraic Riccati equation}

\[ (A - BS^{-1}D^T C)^T X + X(A - BS^{-1}D^T C) - XB^{-1}B^TX + C^TR^{-1}C = 0 \]  

(11)

Where, A, B, C, and D are state space matrices of G, S = I + D^T D \text{ and } R = I + DD^T. Notice that formulae simplify considerably for a strictly proper plant i.e. when D=0.

Then, a minimal state space realization of the normalized left co-prime factorization is given by [22]

\[
\begin{bmatrix}
N & M \\
\end{bmatrix} =
\begin{bmatrix}
A + HC & B + HD & H \\
R^{\frac{1}{2}}C & R^{\frac{1}{2}}D & R^{\frac{1}{2}} \\
\end{bmatrix}
\]  

(12)

Where \( H = -(DD^T)R^{\frac{1}{2}} \).

B. Controller Structure Selection

Assume that \( K(p) \) is structure specified controller. The structure of controller is specified before starting the optimization process. A set of controller parameters \( p \) is evaluated to maximize/minimize objective function. The controller structure is taken as vector \( p \) of the controller parameters is given by \( p = \{ K_p, K_i \} \). The Controller \( K(p) \) can be written as:

\[ K(p) = W_1 K_\infty W_2 \]  

(13)

Assume that \( W_1 \) and \( W_2 \) are invertible, then

\[ K_\infty = W_1^{-1}K(p)W_2^{-1} \]  

(14)

\( W_2 \) is chosen as identity which employees that sensor noise is negligible, then

\[ K_\infty = W_1^{-1}K(p) \]  

(15)

The infinity norm of transfer function \( T_{\infty} \) can be written as

\[
\gamma = \left\| T_{\infty} \right\|_{\infty} = \frac{1}{K_\infty} \left( \left( I + G_{\infty}K_\infty \right)^{-1}M \right)^{-1} \]  

(16)

In this paper evolutionary techniques particle swarm has been adopted to find the optimal values of controller parameters \( p^* \) in stabilizing controller \( K(p) \) such that the \( \left\| T_{\infty} \right\|_{\infty} \) is minimized.

III. CONVENTIONAL LOOP SHAPING DESIGN

According to conventional \( H_{\infty} \) loop shaping design procedure, following steps can be applied:

1) Shape singular values of the nominal plant \( G_0 \) with frequency dependent pre- and post-compensators \( W_1 \) and \( W_2 \) such that the achieved loop-shape i.e. the singular values of the shaped plant \( G = W_1 G_0 W_2 \) satisfies the closed-loop performance requirements. For selecting the weights some guide lines available in [23]. Typically weights are selected such that the open loop plant has the following properties:

- for achieving good performance tracking, good disturbance rejection require , large open loop gain as a rule at low frequency range is necessary
- to achieve good robust stability and sensor nose rejection require a small open loop gain as a rule at high frequency range is necessary
- to achieve the desired bandwidth of controller, selection of crossover frequency (a frequency at which the open loop of the shaped plant intersect with the 0dB line) is important. We select crossover frequency to achieve the desired bandwidth of our controller.

The weighting functions are chosen as:

\[ W_i = K \frac{s + \alpha}{s + \beta} \]  

(17)

Where \( K, \alpha, \beta \) are positive numbers, \( \beta \) is selected as small number (\( \approx 1 \)) for integral action.

The shaped plant is shown in Fig.4

Figure 4. Shaped Plant

2) Minimize the \( H_{\infty} \) -norm of transfer function matrix, \( T_{\infty} \) to find the overall stabilizing controller \( K \).

\[
\gamma = \inf_{\infty} \left\| \frac{1}{K_\infty} \left( I + G_{\infty}K_\infty \right)^{-1}M \right\|_{\infty} 
\]  

(18)

As the resulting optimal gamma is the measure of robustness of desired loop shape, if optimal gamma is less than 4, it means that the designed loop shape is incompatible with robust stability and hence designer should go back to step 1 to design better loop shape (readjust weights).
The gamma optimal can be determined by procedure explained in [24].

\[ \gamma = \varepsilon_{\text{max}}^{-1} = \sqrt{J + \rho(XZ)} \]  
(19)

Where \(X\) and \(Z\) are solution of two algebraic Riccati equations and \(\rho\) represent maximum Eigen value

3) Synthesize a controller \(K_{\text{o}}\) that satisfies negative feedback, achieves a robust stability margin that is slightly less than the computed. The reason for this is that optimal controllers cannot be written in observer form, which is often desirable, and do not roll-off at high frequency (select \(\varepsilon\) less than \(\varepsilon_{\text{optimal}}\)).

Controller \(K_{\text{o}}\) is obtained by solving the control problem in Eq. (21)

\[ \|P_{\text{eq}}\|_e = \left\| \begin{bmatrix} I & G_1 K_{\text{o}} \end{bmatrix}^{-1} M^{-1} \right\|_o \]  
(20)

4) The final controller \(K_{\text{final}}\) is constructed by multiplying the \(K_{\text{o}}\), with weighting functions \(W_1\) and \(W_2\) as depicted in Fig. 5.

\[ K(s)_{\text{final}} = W_1 K_{\text{o}} W_2 \]  
(22)

**Figure 5. Final Controller**

A controller \(K_{\text{o}}\) stabilizes the original closed loop system and minimizes gamma [20].

\[ \gamma = \inf_{k} \left\| \begin{bmatrix} I & G_1 K_{\text{o}} \end{bmatrix}^{-1} M^{-1} \right\|_o \]  
(23)

A. Weight Selection

For selecting the weights, some guide lines are available in [17]. The weighting functions are chosen as:

\[ W_1 = K_1 s + \alpha \frac{s}{s + \beta} \]  
(24)

Where \(K_1, \alpha,\) and \(\beta\) are positive numbers, \(\beta\) is selected as small number (<< 1) for integral action.

**IV. Design of Fixed Order Controller using Particle Swarm Optimization**

In PSO, the potential solution called particles fly through the problem space by following the current optimum particles with an adoptable velocity that is dynamically modified according to its own flying experience and also the flying experience of other particles. Further each particle has a memory and has capable of remembering the best position in the search space ever visited by it [9].

The position corresponding to the best fitness is known as \(P_{\text{best}}\) and the overall best out of all particles in the population called \(g_{\text{best}}\).

\[ V_id = w \times V_id + c_1 \times \text{Rand()} \times (P_id - x_id) + c_2 \times \text{Rand()} \times (g_id - x_id) \]  
(25)

\[ x_id = x_id + V_id \]  
(26)

Particle swarm optimization has many advantages over other conventional optimization techniques:

- PSO is almost derivative free.
- PSO has flexibility to be incorporated with other optimization techniques.
- PSO has fewer parameters to adjust.
- PSO is comparatively easy to put into operations, such as logical operations
- PSO has ability to escape local mini-ma
- PSO does not need an initial solution to begin its process of iterations.

The model of the plant is given in Eq. (28):

\[ G(s) = \frac{551.3}{(s^2 + 43.26s + 536.9)} \]  
(28)

The \(p\) controller structure is taken as vector \(p\) of the controller parameters is given by the \(p = [k_p \ k_i]\). From Eq.(14) Controller \(K(p)\) can be written as:

\[ K(p) = W_1 K_{\text{o}} W_2 \]  
(29)

It is assumed that \(W_1\) is invertible and \(W_2 = 1\)

\[ K_{\text{o}} = W_1^{-1} K(p) \]  
(30)

By substituting Eq. (30) into Eq. (21), we have

\[ J_{\text{out}} = \left\| \begin{bmatrix} I & G_1 W_1^{-1} K(p)^{-1} (I - G_1) \end{bmatrix} \right\| \]  
(31)

A. Sequential scheme for proposed approach
The steps for the sequential scheme for proposed approach can be described as follows:

Step1. Shape the singular values of the nominal plant by selecting \( W_1 \) and \( W_2 \), then calculate the gamma using Eq. (19). If gamma is greater than 4, it shows that weighting functions are incompatible; the weight \( W_1 \) will be adjusted.

Step2. Initializes several sets of population parameters \( p \) as population of particles, where \( p \) is considered as a vector of controller parameters.

Step3. Specify the controller structure; evaluate the cost function of each particle by using Eq. (31) according to cost function.

Step4. At each generation the velocity of each particle is calculated by using Eq. (25) and positions of next is according to Eq. (26).

Step5. If current iteration is less than maximum iterations then stop, go to step 3. Finally, checks the performance in both frequency and time domain. The flow chart of the proposed scheme as shown in Fig. 5:

\[
G(s) = \frac{413.5s + 2205e^{-0.125s}}{s^3 + 43.26s^2 + 536.9s + 0.5369} \quad (33)
\]

The stabilizing controller \( K_\infty \) can be obtained by Matlab Code,

\[
K_\infty(s) = \frac{413.5s + 2205}{s^3 + 43.26s^2 + 536.9s + 0.5369} \quad (34)
\]

By putting Eq. (34) and Eq. (32) in Eq. (22), then \( K(s)_{\text{final}} \) can be obtained as:

\[
K(s)_{\text{final}} = \frac{310.1s^2 + 3308s + 8221}{s^4 + 43.26s^3 + 537s^2 + 1.074s + 0.05369} \quad (35)
\]

The controller obtained by conventional techniques Eq. (35) is of 4th order and its structure is complex as well, obviously it is difficult to implement practically for industrial application. Hence the advantage of fixed order can be obtained from proposed approach. A PI controller is investigated as a fixed order controller, \( K_p \) and \( K_i \) are the controller parameters that would be evaluated by using PSO. The controller structure is expressed in Eq. (36)

\[
K(p) = K_p + \frac{K_i}{p} \quad (36)
\]

The simulation was carried out using representation of particles. The size of initial population is 10 particles. Algorithm converged in second iterations, and gave optimal cost function of 1.395. Fig. 7 shows the plot of convergence of cost function versus iterations of PSO.

\[
K(p)^* = 0.400 + \frac{1.00}{p} \quad (37)
\]

Evidently, the controller obtained by conventional techniques is of high order and its structure is very complex, obviously difficult to implement practically. Thus the controller is Eq. (35) is 4th order controller and has complex structure. Hence the advantage of fixed structure can be obtained from the proposed techniques.
VI. Conclusion

In this paper, a new approach for designing a fixed order robust controller is proposed. In the proposed design approach controller parameter optimization and minimizing the cost function is based on PSO. The performance of control system using controller designed by using the proposed approach was tested for closed loop step response. The simulation results show that the algorithm converges and system has satisfactory step response.

Moreover, in cases where complex mathematical calculations, algebraic optimization and fraction of polynomial matrices are involved, PSO is a better alternative to solve an optimization problem. The proposed approach will enable the practicing engineers to employ the approach for designing of controller with low, fixed order, like PID controllers, which have high acceptances in industrial applications.

References


Author Biographies

Faizullah Mahar received his B.E from Mehran University of Engineering and Technology Jamshoro, in 1988 and MS degree from NED University of Engineering and Technology, Karachi, Pakistan, in 1995. Since April, 1992, he has been working as an Assistant Professor in the Department of Electrical Engineering, Baluchistan University of Engineering and technology, Khuzdar, Pakistan. From spring 2006 he is Ph.D. scholar at the Iqra University, Karachi Pakistan. He is the author of the more than fifty research articles. His research interests include control systems, robust control system and applications of modern evolutionary optimization techniques.

Syed Saad Azhar Ali received his B.E from NED University of Engineering and Technology, Karachi, Pakistan, in 1999. He received MS and PhD degrees in Electrical engineering from the King Fahd University of Petroleum and Minerals, KSA, in 2001 and 2007 respectively. He author of three books. In 2009 he joined the Department of Electronics Engineering, where he is an Associate Professor. His research interests are adaptive control, intelligent control system and signal processing.

Ayaz Hussain is an Assistant Professor in the Department of Electrical Engineering at Balochistan University of Engineering and Technology, Khuzdar Pakistan. He received the Bachelor degree in Telecommunication Engineering from the Mehran University of Engineering and Technology, Jamshoro, Pakistan (2006) and MS-Engineering degree in Electrical from the Hanyang University, Ansan, South Korea (2010). His current research interests include the control Systems and wireless communication, with emphasis on cooperative relay system.