International Journal of Computer Information Systems and Industrial Management Applications.
ISSN 2150-7988 Volume 5 (2013) pp. 308-316

Cross Entropy Optimization for Optimal Design of Water Distribution Networks

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***Abstract*: This paper presents cross entropy (CE) based methodology for optimal design of water distribution network (WDN). Design of WDN involves selection of suitable diameter for each pipe in the network from the list of commercially available diameters. The CE methodology is applied to two bench mark WDN design problems taken from literature for validation. The first WDN problem deals with determining optimal pipe sizes for planning a new system, while the second WDN deals with rehabilitation of existing WDN by parallel piping. The performance of CE is compared with the results of past studies and it is found that the CE resulted in good optimal solutions. Then, the model is applied to a case study in India. The results suggest that CE method is very effective in optimal design of water distribution networks and has the capability of rapid convergence to optimum solutions.**

Keywords*:* water distribution networks, metaheuristics, cross entropy, optimization.

I. Introduction

Water distribution network (WDN) consists of a set of pipes of different diameters and lengths connected with one another at various junctions called nodes. The diameters and lengths of pipes are designed in such a manner that they deliver the required amount of water with sufficient pressure to the demand nodes without failure. The optimal design of WDN aims to find a combination of the diameters that are feasible and results in minimum cost. Several researchers have formulated different models for optimal design of WDNs. Few studies modeled it as a nonlinear model and solved using Nonlinear Programming (NLP) techniques by treating discrete pipe sizes as continuous variables. The main disadvantage of these NLP methods is the required rounding-off of continuous decision variables to commercially available sizes, sometimes which can lead to network infeasibilities as well as raise questions on optimality of the adjusted solution. Some other studies formulated linear models and solved it using Linear Programming (LP) techniques. However, these methods are capable of maintaining the constraint on discrete pipe sizes (without the need for rounding off solutions), but it requires approximation of non-linear functions, which may not represent the reality as it is.

In spite of development of many conventional techniques for optimization, each of these techniques has its own limitations. To overcome those limitations, recently metaheuristic techniques are being used for solving combinatorial optimization problems. By using these techniques, the given problem can be represented more realistically. These also provide ease in handling the non-linear relationships of the formulated model [1]. Genetic algorithms, particle swarm optimization, ant colony optimization algorithm, cross entropy algorithm etc. are some of the techniques fall in this category. These evolutionary algorithms search from a population of points, so there is a greater possibility to cover the whole search space and locating the global optimum.

The stochastic search approaches that were used for WDN design include genetic algorithms ([2]; [3]; [4]; [5]; [1]; [6]), Simulated annealing [7], shuffled leaping frog algorithms [8], ant colony optimization algorithms [9], cross entropy algorithms [10] etc. These techniques improve the quality of the solution over the iterations by using heuristics.

The cross entropy (CE) method was motivated by an adaptive algorithm for estimating probabilities of rare events in complex stochastic networks, which involves variance minimization. Later it was modified to a randomized optimization technique, where the original variance minimization was changed to cross entropy minimization problem [11]. The CE method was successfully applied to various optimization problems such as traveling salesman, assignment problem etc.[11]. In the present study, CE method is presented for optimization of the water distribution networks.

II. Cross Entropy Method

The cross entropy method is an iterative technique based on the concept of rare events, which involves two main stages: (i) generation of random sample of initial population (i.e., solution vectors) with a set of parameters, and (ii) updating this set of parameters which control the generation of random data using the sample itself, with the aim of improving the solution in the next iteration. The method derives its name from the cross entropy or Kullback-Leibler distance- a well known measure of 'information', which has been successfully employed in various fields of engineering [11].

A. Entropy and Cross Entropy

Entropy can be termed as a measure of uncertainty associated with a process (measure of expected information gain from a random variable) [12]. The probability distribution of events if known provides a certain amount of information. Shannon defined a quantitative measure of the distribution in terms of entropy, called Shannon entropy given by (1).

$$H(X) = -K\sum_{r=1}^{n} p_r \ln p_r \tag{1}$$

where $H(X)$ represents the Shannon entropy corresponding to the random variable X, K is a constant, and p_r represents the discrete probability corresponding to the variable at x_r. The uncertainty can be quantified with entropy taking into account all different kinds of available information. Thus entropy is a measure of uncertainty represented by the probability distribution and is a measure of the lack of information about a system. If complete information is available, entropy is equal to zero, otherwise it is greater than zero.

Cross entropy is a distance measure from one probability distribution to another. One of the well known definitions of Cross entropy is the Kullback–Leibler distance measure [13], serving to assess the similarity between two probability distributions: the assumed distribution $q(x)$ and the actual distribution $p(x)$. Cross entropy [$D(P,Q)$] is formulated as in (2).

$$D(P,Q) = \sum_{r=1}^{n} p_r \ln \frac{p_r}{q_r} \tag{2}$$

The interpretation of (2) is that in order to estimate a probability distribution, the cross entropy should be minimized. The goal is to find a distribution $p(x)$ for which the Kullback – Leibler distance between $p(x)^*$ and $q(x)^*$ is minimal.

B. Principle of Minimum Cross Entropy

According to Laplace's principle of insufficient reason, all outcomes of an experiment should be considered equally likely unless there is information to the contrary [13]. Suppose a probability distribution for a random variable $X = X = \{x_1, x_2, x_3,\ldots,x_n\}$ is assumed as $Q = \{q_1, q_2, q_3,\ldots,q_n\}$ based on intuition. This constitutes the prior information in terms of a prior distribution. While estimating the actual distribution $P = \{p_1, p_2, p_3,\ldots,p_n\}$ of random variable X, using all the given information and make the distribution as near as possible to the assumed distribution. Thus, according to the principle of minimum cross entropy (POMCE), the cross entropy (CE), $D(P,Q)$ is minimized as in (3).

$$\text{Minimize } D(P,Q) = \sum_{r=1}^{n} p_r \ln \frac{p_r}{q_r} \tag{3}$$

This is referred to as the principle of minimum cross entropy, which minimizes the Bayesian entropy [13]. Here minimizing $D(P,Q)$ is equivalent to maximizing the Shannon entropy.

C. Cross Entropy Algorithm

The main steps involved in the cross entropy algorithm for solving combinatorial optimization problem is given below.

1. Conversion of the combinatorial optimization problem to a stochastic node network (SNN) problem.
2. Set the trial counter $t = 0$ and assume equal probabilities for all the options as $p_{0,r}$, where 'r' takes values from 1 to m. The number of stochastic nodes, $m=n_p*nd$, where n_p is the number of variables and nd is the number of available options.
3. Generate N_c sample vectors $X_v(x_1, x_2,\ldots,x_m)$ for $v = 1$ to N_c using the probability $p_{t,r}$ (*i.e.*, generate a set of N_c possible vectors each of size m, and having zeros and ones, where one corresponds to choosing a specific node, and zero otherwise). The value of N_c is taken as $N_c = \beta*nd$, where β is an integer value. The m dimensional vector X_v $(x_1, x_2,\ldots,x_m)$ has the discrete probability of $P = (p_1, p_2,\ldots,p_m)$.
4. Find out the performance function $S(X_v)$ and check for constraints corresponding to each of the random vectors X_v, generated.
5. Now arrange the random vectors X_v, in the ascending order(if the problem is a minimization problem) or descending order (if it is a maximization problem) of their performance function $S(X_v)$ values. Now the top most vectors will be having the best performance value and it is denoted as γ_t.
6. Choose a set (say ρ_c) of the top best performing vectors for updating the probability vector $p_{t,r}$ to the probability vector $p_{t+1,r}$. Here ρ_c corresponds to percentage of the vectors selected and its value varies between 10% and 20% but may change as a function of the sample size N. The r^{th} component of $p_{t+1,r}$ is obtained as given by (4).

$$p_{t+1,r} = \frac{B_{t,r}}{TB_t} \tag{4}$$

where $p_{t+1,r}$ is the probability of success in the $(t+1)^{\text{th}}$ iteration of node r, $B_{t,r}$ is the total number of times node r was chosen (frequency) out of the best top performance vectors (i.e., TB_t the total number of vectors in the elite set) at iteration t.

In order to avoid early convergence (stopping criteria of probabilities of potential options approaching ZERO or ONE) to a local optimum solution, a smoothing parameter (α_c) is used. The probability is modified as given by (5)

$$p_{t+1,r} \leftarrow \alpha_c p_{t+1,r} + (1-\alpha_c) p_{t,r} \tag{5}$$

Using the above probability-updating scheme, the probability of choosing a node at each subsequent iteration increases as the frequency of occurrence of the node in the elite set increases. Updating the entire probability components using (4) in conjunction with the smoothing formula (5) yields the new probability vector $p_{t+1,r}$. The main reason why such a smoothing updating procedure performs better is that it prevents the incidents of zeroes and ones in the reference vector, as in case such values are obtained they will remain permanently, which is obviously not required.

7. Check stopping conditions: If γ_t for subsequent iterations remains unchanged and if p_t converges to the degenerated case (*i.e.* all the probabilities $p_{t,r}$ are close to zero and one) then stop. Declare the last γ_t as the optimal solution γ^* and its associated vector X as the design vector X^*, otherwise $p_{t,r} \leftarrow p_{t+1,r}$ and return to step 3.

III. Model Formulation

The optimization problem is to determine the values of pipe diameters that would minimize the cost of the system without violating any of the constraints. Thus it is required to select one diameter for each pipe from the list of commercially available diameters. The optimization problem can be expressed as,

$$Minimize\ Cost = \sum_{i=1}^{n_p} C(d_i) * l_i \tag{6}$$

subject to,

$$H_j \geq H_j^{\min}, \quad \forall\ j \tag{7}$$

$$q_j^{in} - q_j^{out} - q_j = 0\ ,\ \ \forall j \tag{8}$$

$$\left(\sum_{i=1}^{np_L} HL_i - \sum_{p=1}^{npu_L} h_p \right)_L = 0\ ,\quad L = 1,2,3,.......,nL \tag{9}$$

where,

$$q_j^{in} = \sum_{i=1}^{n_{in}} Q_i \tag{10}$$

$$q_j^{out} = \sum_{i=1}^{n_{out}} Q_i \tag{11}$$

$$HL_i = \frac{\delta l_i Q_i^{1.852}}{C_{HW}^{1.852} d_i^{4.87}} \tag{12}$$

where $C(d_i)$ corresponds to the cost per unit length of the pipe having diameter d_i and l_i is the length of the i^{th} pipe , H_j and $H_j^{\min}$ are the available and minimum pressure heads at the j^{th} node; n_d =number of demand nodes; q_j^{in} = flow entering the j^{th} node; q_j^{out} =flow leaving from the j^{th} node ; q_j= demand at the j^{th} node; HL_i =head loss in i^{th} pipe; np_L=number of pipes in a loop; h_p =head raised by the pump p, npu_L=number of pumps in a loop; nL=number of loops in the WDN. n_{in} =number of incoming pipes to the j^{th} node; n_{out} =number of outgoing pipes from the j^{th} node; and Q_i = discharge or flow through the i^{th} pipe, δ =constant depending on the units of head loss, length, diameter, and discharge; and C_{HW}=Hazen William's roughness coefficient.

IV. Application of the Model

A. *Case Study I: Hanoi WDN*

The Hanoi water distribution network problem [14] as shown in Figure 1, is an extensively studied WDN by many researchers using a variety of optimization methods (such as genetic algorithms, ant colony optimization, simulated annealing etc.) is taken-up as case study I for testing the performance of CE method. This network is a real WDN constructed in Hanoi city at Vietnam, consists of 34 pipes and 32 nodes organized in three loops. The system is gravity fed by a single reservoir which is located at elevation of 100 *m*. The ground elevation for all nodes is 0. All pipes in the network are of different lengths and the length of pipes is given in Table 1. Data relevant to nodes is given in Table 2. The system constraint on minimum pressure head requirement for all nodes is defined as 30 *m*. No velocity constraint is taken into account for this network. There are 6 commercially available pipe diameters (*nd*=6) and unit cost of the pipes used in the case study I are given in Table 3.

The study on Hanoi WDN was first carried out by [14]. Thereafter so many researchers [4], [7], [15] and [16] applied various techniques to find optimal solution to Hanoi WDN. The solution search space for the Hanoi WDN is 6^{34}.

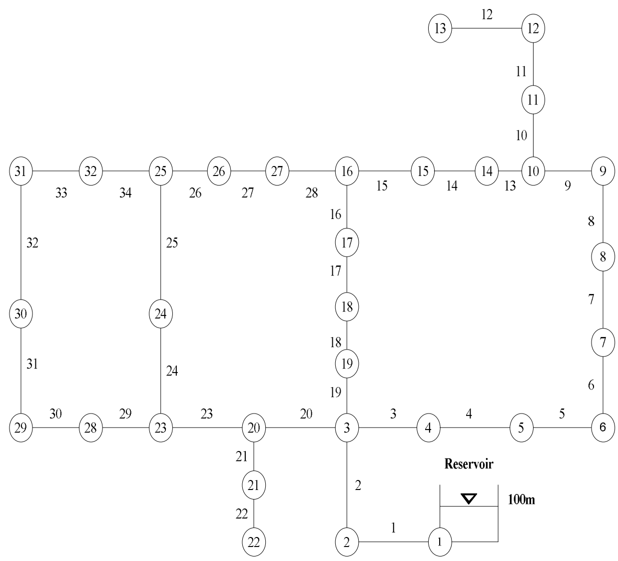

Figure 1. Layout of Hanoi WDN

Table 1. Pipe length data for Hanoi WDN

Pipe No.	Pipe Length (*m*)	Pipe No.	Pipe Length (*m*)
1	100	18	800
2	1350	19	400
3	900	20	2200
4	1150	21	1500
5	1450	22	500
6	450	23	2650
7	850	24	1230
8	850	25	1300
9	800	26	850
10	950	27	300
11	1200	28	750
12	3500	29	1500
13	800	30	2000
14	500	31	1600
15	550	32	150
16	2730	33	860
17	1750	34	950

Table 2. Node demand data for Hanoi WDN

Node No.	Nodal Demand (m^3/h)	Node No.	Nodal Demand (m^3/h)
1	-	17	865
2	890	18	1345
3	850	19	60
4	130	20	1275
5	725	21	930
6	1005	22	485
7	1350	23	1045
8	550	24	820
9	525	25	170
10	525	26	900
11	500	27	370
12	560	28	290
13	940	29	360
14	615	30	360
15	280	31	105
16	310	32	805

Table 3. Commercially available pipe diameters and unit cost of pipes for Hanoi WDN

Sl. No.	Available Pipe Diameter		Unit Cost of Pipe ($/*m* length)
	inch	*mm*	
1	12	304.8	45.73
2	16	406.4	70.40
3	20	508	98.38
4	24	609.6	129.30
5	30	762.0	180.75
6	40	1016.0	278.28

1) Model Run and Output for Case Study I

At the start of the algorithm, it is assumed that all the options have equal probability of selection (i.e., $P_{0,r} = 1/6$).

The performance function used for solving the model is

$$S(X_v) = \sum_{i=1}^{n_p} C(d_i) \times l_i + \sum_{j=1}^{n_n} PN \times MAX\,(0, H_j^{\min} - H_j) \qquad (13)$$

where $S(X_v)$ is the performance function for the solution vector, and *PN* is the penalty function rate for violating the nodal pressure constraint.

At the end of the final iteration, only 34 (i.e., total number of pipes) options will be having probability equal to one which forms the optimal solution set, and rest of them will be having a probability equal to zero. A program in MATLAB is developed, which is linked to EPANET toolkit for simulation of the WDN and to check the hydraulic feasibility. The stopping criteria is arrived in 31,500 function evaluations with smoothing parameter α = 0.35 and *PN* =100000000. The output of the model run for Hanoi WDN is given in Tables 4 & 5, and also compared with the past studies.

Table 4. Nodal pressure corresponding to the optimal design by Cross Entropy method for Hanoi WDN

Node No.	Available Nodal Pressure (*m*)	Remarks	Node No.	Available Nodal Pressure (*m*)	Remarks
1(R)	100	Reservoir	17	32.9603	
2	97.1407		18	49.8247	
3	61.6704		19	55.0349	
4	57.1713		20	50.0175	
5	51.5992		21	40.6683	
6	45.7571	Avail. pressure is more than the min. pressure required	22	39.3963	Avail. pressure is more than the min. pressure required
7	44.4013		23	43.4291	
8	42.8160		24	37.5807	
9	41.5661		25	33.7794	
10	40.6585		26	31.7037	
11	39.0991		27	30.9604	
12	35.6707		28	35.1562	
13	31.4625		29	30.7902	
14	33.3626		30	30.1112	
15	30.5197		31	30.6475	
16	30.4795		32	32.0296	

Table 5. Comparison of Cross Entropy model result of Hanoi WDN with past studies

Pipe No.	Pipe Diameter (*inch*) as per:		
	[4]	*[15]*	*Cross Entropy Method*
1	40	40	40
2	40	40	40
3	40	40	40
4	40	40	40
5	40	40	40
6	40	40	40
7	40	40	40
8	40	40	40
9	30	40	40
10	30	30	30
11	30	24	24
12	24	24	24
13	16	24	16
14	16	12	12
15	12	12	16
16	16	12	12
17	20	16	16
18	24	24	24
19	24	24	20
20	40	40	40
21	20	20	20
22	12	12	16
23	40	40	40
24	30	30	30
25	30	30	30
26	20	20	24
27	12	12	16
28	12	12	12
29	16	16	16
30	16	12	16
31	12	12	12
32	12	20	12
33	16	16	16
34	20	24	24
Total cost ($)	6.18×10^6	6.11×10^6	6.15×10^6

On comparing the results of the Cross Entropy Model for Hanoi WDN problem [14] with results of earlier studies, it is found that the optimum diameters obtained from the present study is coming nearly same for all pipes in the network except for few pipes. Also the optimal cost obtained is closer to optimal costs of previous studies. Thus, the results obtained from present study shows that the CE method is effective and is well suited for optimal design of medium sized WDN like Hanoi WDN.

B. *Case Study II: Newyork City Tunnel WDN*

Newyork City Tunnel WDN [15] is taken-up as a case study II, for testing the performance of CE method. The layout of WDN is shown in Figure 2. The network consists of 20 nodes, 21 pipes and 1 loop, and is fed by gravity from a reservoir at a fixed head of 300 *ft* (91.44 *m*). The ground elevation for all nodes is 0. This system is in place and requires expansion. The pipe lengths, existing pipe diameters, and nodal demands are given in Table 6, and a Hazen-Williams constant of 100 is assumed for both the old tunnels and new pipes [15]. The system constraint is the minimum pressure head requirement for all nodes which is also given in Table 6. Fifteen commercially available pipe diameters and their unit cost are listed in Table 7. No velocity constraint is taken into account for this network. The objective is to determine whether a new pipe is to be laid parallel to an existing pipe or not, and if needed what will be the diameter of a parallel pipe, while the system is required to provide minimum hydraulic gradients. This network is firstly studied in [17] and thereafter studied by a number of other researchers ([4]; [5]; [6]). Due to pipe aging, the existing gravity flow tunnels are inadequate to meet the pressure requirements at nodes 16, 17, 18, 19, and 20 for the projected demands. Therefore new pipes can be added in parallel to the existing pipes to meet the minimum pressure head requirements. For this problem, 16 possible candidate diameters are available including 15 commercially available diameters and the 'zero diameter-zero unit cost' option. Considering all 21 pipes for possible duplication, it results in 16^{21} possible designs.

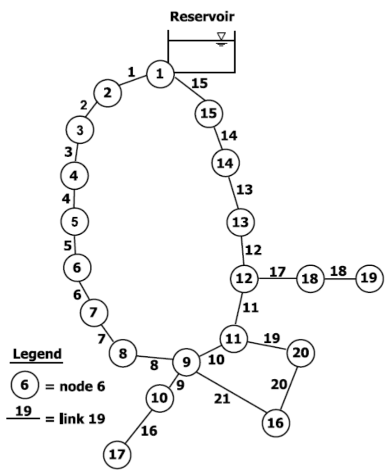

Figure 2. Layout of Newyork City Tunnel WDN

Table 6. Data for Newyork city tunnel WDN

Pipe No.	Pipe Length (*m*)	Existing Pipe Diameter (*inch*)	Node No.	Nodal Demand (m^3/h)	Minimum Required Nodal Pressure (*m*)
1	3536.59	180	1	-205665	91.44
2	6036.59	180	2	9419.317	77.72
3	2225.61	180	3	9419.317	77.72
4	2530.49	180	4	8991.166	77.72
5	2621.95	180	5	8991.166	77.72
6	5823.17	180	6	8991.166	77.72
7	2926.83	132	7	8991.166	77.72
8	3810.98	132	8	8991.166	77.72
9	2926.83	180	9	17329.91	77.72
10	3414.63	204	10	101.941	77.72
11	4420.73	204	11	17329.91	77.72
12	3719.51	204	12	11937.25	77.72
13	7347.56	204	13	11937.25	77.72
14	6432.93	204	14	9419.317	77.72
15	4725.61	204	15	9419.317	77.72
16	8048.78	72	16	17329.91	79.25
17	9512.2	72	17	5861.588	83.15
18	7317.07	60	18	11937.25	77.72
19	4390.24	60	19	11937.25	77.72
20	11707.32	60	20	17329.91	77.72
21	8048.78	72			

Table 7. Commercially available pipe diameters and unit cost of pipe for Newyork city tunnel WDN

Sl.No.	Pipe Diameter		Unit Cost of Pipe	
	(*inch*)	(*mm*)	(*$/foot*)	(*$/metre*)
1	36	914.4	93.5	306.7
2	48	1219.2	134	439.6
3	60	1524	176	577.4
4	72	1828.8	221	725
5	84	2133.6	267	875.9
6	96	2438.4	316	1036.7
7	108	2743.2	365	1197.5
8	120	3048	417	1368.1
9	132	3352.8	469	1538.7
10	144	3657.6	522	1712.6
11	156	3962.4	577	1893
12	168	4267.2	632	2073.4
13	180	4572	689	2260.5
14	192	4876.8	746	2447.5
15	204	5181.6	804	2637.7

1) *Model Run and Output for Case Study II*

The 21 existing pipes are considered as such and 21 parallel pipes for all the 21 pipes with 16 candidate diameters. At the start of the iteration, it is assumed that all the potential alternatives have equal probability of selection (i.e.,$P_{0,r}$=1/16), since there are 16 candidate diameters including 'zero diameter- zero unit cost' option. While using

EPANET, to avoid problems with consideration of zero diameter pipes, negligibly small diameter (i.e., 0.0001 *mm*) with zero unit cost is considered. The performance function used for solving the model is given by (13).

As the iteration begins, some of the candidate diameters becomes superior to the others based on the performance values and their probability increases while for others gets reduced. This step by step iterative procedure for the modification to the probability of candidate diameters will continue until they reach the stopping criteria of approximately ones and zeros in the final iteration. At the end of the final iteration, only 21(i.e., total number of pipes) candidate diameters will be having probability equal to one which forms the optimal solution set, and all the rest will be having a probability equal to zero. The stopping criteria is arrived in 36,000 function evaluations with smoothing parameter α = 0.35 and *PN* =10000000. The output of the model run for Newyork City Tunnel WDN is given in Table 8 and also compared with the past studies, which is given in Table 9.

Table 8. Cross Entropy Model output for Newyork city tunnel WDN

Pipe No.	Pipe Length (*m*)	Existing Pipe Diameter (*inch*)	Parallel Pipe Diameter (*inch*)	Node No.	Available Nodal Pressure (*m*)	Minimum nodal Pressure required (*m*)
1	3536.59	180	0	1	91.44	91.44
2	6036.59	180	0	2	89.6743	77.72
3	2225.61	180	0	3	87.2179	77.72
4	2530.49	180	0	4	86.4983	77.72
5	2621.95	180	0	5	85.861	77.72
6	5823.17	180	0	6	85.3664	77.72
7	2926.83	132	144	7	84.5863	77.72
8	3810.98	132	0	8	84.328	77.72
9	2926.83	180	0	9	83.4469	77.72
10	3414.63	204	0	10	83.4373	77.72
11	4420.73	204	0	11	83.4745	77.72
12	3719.51	204	0	12	83.8627	77.72
13	7347.56	204	0	13	84.7651	77.72
14	6432.93	204	0	14	87.04	77.72
15	4725.61	204	0	15	89.4058	77.72
16	8048.78	72	96	16	79.2747	79.25
17	9512.2	72	96	17	83.1702	83.15
18	7317.07	60	84	18	79.6084	77.72
19	4390.24	60	72	19	77.7403	77.72
20	11707.3	60	0	20	79.4684	77.72
21	8048.78	72	72			

The optimal solution obtained in the present study by using the Cross entropy method is satisfying the minimum pressure head requirement at all the nodes, and is resulting in minimum cost.

Table 9. Comparison of optimal outputs obtained by various approaches for Newyork city tunnel WDN

Pipe No.	Existing Pipe Diameter (*inch*)	Diameter of parallel pipe (*inch*)		
		Improved GA [5]	***Messy GA [6]***	***Cross Entropy Method***
1	180	0	0	0
2	180	0	0	0
3	180	0	0	0
4	180	0	0	0
5	180	0	0	0
6	180	0	0	0
7	132	0	144	144
8	132	0	0	0
9	180	0	0	0
10	204	0	0	0
11	204	0	0	0
12	204	0	0	0
13	204	0	0	0
14	204	0	0	0
15	204	120	0	0
16	72	84	96	96
17	72	96	96	96
18	60	84	84	84
19	60	72	72	72
20	60	0	0	0
21	72	72	72	72
Cost (in *million $*)		38.8	38.64	38.64
No. of function evaluations		96,750	37,186	36,000
Feasibility		Feasible	Feasible	Feasible

On comparing the results of the CE method for Newyork City Tunnel WDN with the results of past studies on the same WDN, it is found that the number of parallel pipes to be added is nearly same with only slight difference in one of the parallel pipe diameters with approximately same cost for providing parallel pipes. The number of function evaluations taken for producing the optimum solution is less than the other approaches. Thus, the results obtained from present study shows that the cross entropy method is effective and is well suited for the optimal design of large network like Newyork City Tunnel WDN, which involves capacity expansion in terms of adding parallel pipes without disturbing the existing pipes.

C. Case Study III: Bengali Camp Zone WDN

The Bengali Camp Zone WDN of Chandrapur city in Maharashtra State, India is taken as a case study III. This is a real WDN of Chandrapur water supply system, and whose network details are shown in Figure 3.

This WDN is built to serve a new residential area in the city. The network was designed as an extension to the original WDN of Chandrapur city. The projected population for the year 2040 of Bengali Camp zone and the peak factor adopted for the design of WDN are 47126 and 3 respectively. The

Bengali Camp zone WDN consists of 34 nodes, 38 pipes, and is fed by gravity from a tank at a fixed head of 206 *m*. The existing pipe diameters, and nodal pressure for the Bengali Camp zone WDN are given in Table 10. The pipe details and lengths are given in Table 11; nodal elevations and nodal demands are given in Table 12. A Hazen-Williams constant of 140 is assumed for all the pipes. The system constraint is minimum pressure requirement for all nodes is 11 *m*. Twelve commercially available pipe diameters and their unit cost are listed in Table 13. No velocity constraint is taken into account for this network. The solution search space of Bengali Camp zone WDN is 12^{38}.

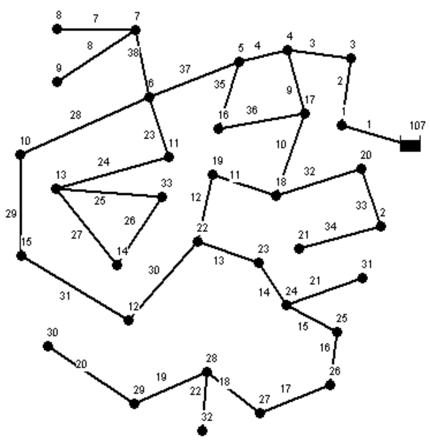

Figure 3. Layout of Bengali Camp Zone WDN

Table 10. Pipe diameters, and nodal pressure as per existing design for Bengali camp zone WDN

Pipe No.	Existing Pipe Diameter (*mm*)	Pipe No.	Existing Pipe Diameter (*mm*)	Node No.	Available Nodal Pressure (*m*)	Node No.	Available Nodal Pressure (*m*)
1	500	20	250	1	10.98	18	18.75
2	500	21	200	2	11.66	19	18.89
3	500	22	150	3	17.33	20	19.29
4	300	23	200	4	18	21	19.64
5	100	24	200	5	18.26	22	20.69
6	100	25	100	6	14.63	23	19.41
7	150	26	100	7	16.43	24	22.48
8	150	27	100	8	20.12	25	24.22
9	450	28	150	9	16.13	26	27.18
10	450	29	150	10	15.03	27	25.11
11	450	30	150	11	14.8	28	24.88
12	450	31	100	12	16.64	29	24.46
13	450	32	150	13	14.17	30	21.94
14	450	33	150	14	16.25	31	21.48
15	400	34	100	15	11.51	32	25.06
16	400	35	100	16	18.27	33	15.95
17	400	36	150	17	17.97	34	18.75
18	350	37	300			107	-
19	300	38	300				
Cost of the WDN as per Existing Design (₹)			25735031				

Table 11. Pipe details for Bengali camp zone WDN

Pipe No.	Start Node	End Node	Pipe Length (*m*)	Pipe No.	Start Node	End Node	Pipe Length (*m*)
1	107	1	61	20	29	30	385
2	1	3	413	21	24	31	275
3	3	4	83	22	28	32	165
4	4	5	165	23	6	11	138
5	5	6	715	24	11	13	248
6	6	7	193	25	13	33	303
7	7	8	413	26	33	14	193
8	7	9	220	27	13	14	330
9	4	17	72	28	6	10	330
10	17	18	77	29	10	15	165
11	18	19	165	30	22	12	770
12	19	22	660	31	15	12	248
13	22	23	330	32	18	20	220
14	23	24	715	33	20	2	275
15	24	25	330	34	2	21	220
16	25	26	248	35	16	5	83
17	26	27	468	36	17	16	165
18	27	28	138	37	5	6	715
19	28	29	715	38	6	7	193

Table 12. Node details for Bengali camp zone WDN

Node No.	Nodal Elevation (*m*)	Base Demand	
		(*LPS*)	(m^3/h)
1	195	0	0
2	194	1.27	4.572
3	188.5	0	0
4	187.8	0.607	2.1852
5	187.5	2.469	8.8884
6	191	3.517	12.6612
7	189.2	2.063	7.4268
8	185.5	1.031	3.7116
9	189.5	0.55	1.98
10	190.5	1.27	4.572
11	190.8	0.989	3.5604
12	188.8	2.61	9.396
13	191.4	2.259	8.1324
14	189.2	1.34	4.824
15	194	1.059	3.8124
16	187.5	0.211	0.7596
17	187.8	0.382	1.3752
18	187	1.186	4.2696
19	186.8	2.116	7.6176
20	186.4	1.27	4.572
21	186	0.564	2.0304
22	184.8	4.515	16.254
23	186	8.903	32.0508
24	182.8	10.055	36.198
25	181	1.91	6.876
26	178	2.364	8.5104
27	180	2.939	10.5804
28	180.2	10.309	37.1124
29	180.5	11.145	40.122
30	183	3.901	14.0436
31	183.8	0.91	3.276
32	180	1.673	6.0228
33	189.5	1.27	4.572
107(Resvr)	195	-	-

Table 13. Commercially available pipe diameters and unit cost of pipe for Bengali camp zone WDN

Sl. No.	Available Pipe Diameter		Unit Cost of Pipe (₹/m length)
	inch	*mm*	
1	4	100	860
2	6	150	1077
3	8	200	1374
4	10	250	1840
5	12	300	2333
6	14	350	2885
7	16	400	3442
8	18	450	4142
9	20	500	4826
10	24	600	6375
11	28	700	8141
12	32	800	10161

1) Model Run and Output for Case Study III

At the start of the run, it is assumed that all the candidate diameters have equal probability of selection (i.e., $P_{0,r}$=1/12). The performance function used for solving the model is given by (13). As the iteration progresses, some of the candidate diameters become superior to the others based on the performance values and their probability increases, while for others the probability gets reduced. This step-by-step iterative procedure for updating the probability of selecting a candidate diameter for each pipe will continue until they satisfy the stopping criteria. At the end, the probability of selecting a option for a pipe will be approximately equal to ones and zeros. This means that only 38 decisions (i.e., total number of pipes) will be having probability equal to one which forms the optimal solution set, and the remaining will be having a probability equal to zero. The stopping criteria is arrived in 38,400 objective function evaluations with smoothing parameter α = 0.35 and $PN = 10^8$. The output of the model run for Bengali Camp Zone WDN is given in Table 14.

Table 14. Cross Entropy Model Output For Bengali Camp Zone WDN

Pipe No.	Optimum Pipe Diameter (*mm*)	Pipe No.	Optimum Pipe Diameter (*mm*)	Node No.	Available Nodal pressure (*m*)	Node No.	Available Nodal pressure (*m*)
1	600	20	200	1	10.99	18	18.89
2	600	21	100	2	11.8	19	19.05
3	600	22	100	3	17.43	20	19.43
4	300	23	150	4	18.19	21	19.78
5	100	24	200	5	18.38	22	20.86
6	150	25	100	6	14.74	23	19.57
7	100	26	100	7	16.52	24	22.55
8	100	27	100	8	20.12	25	24.29
9	500	28	150	9	16.21	26	27.20
10	500	29	150	10	15.11	27	25.13
11	500	30	100	11	14.82	28	24.90
12	450	31	100	12	16.65	29	24.48
13	450	32	150	13	14.19	30	21.94
14	400	33	150	14	16.27	31	21.49
15	400	34	100	15	11.58	32	25.00
16	350	35	100	16	18.38	33	15.97
17	400	36	100	17	18.10	34	18.89
18	350	37	300			107	-
19	300	38	150				
Optimum Cost (₹)			25235630				

On comparing the results of the CE method for Bengali Camp zone WDN with the existing design, it is noticed that the optimal solutions of CE is better than existing design, resulting in 1.94% lesser cost than the existing design. The solution is obtained in 38,400 function evaluations. Also the minimum nodal pressure requirements are well satisfied. The results of present study amply demonstrate that the CE method is an effective optimization method for WDN and has capability to handle larger number of discrete decision variables and various constraints. Thus, CE method is well suited for optimal design of larger water supply networks.

V. Conclusions

This study presented Cross Entropy (CE) method for solving water distribution network optimization problems. For hydraulic simulation of WDNs, EPANET tool kit is adopted and carried out simulation-optimization modeling for design of WDNs. Initially, the CE method is applied for two benchmark WDN design problems, namely Hanoi WDN and Newyork city tunnel WDN. To evaluate the performance of CE optimization method, the results are compared with the past studies and it is found that the CE method is giving good quality optimal solutions in a few number of objective function evaluations. The results also demonstrated that the CE method can be used effectively for optimal design of new WDN as well as for rehabilitation of existing WDN (i.e., for capacity expansion of WDNs, in terms of adding parallel pipes without disturbing the existing pipes). It is also found that the CE method is capable of handling larger number of discrete decision variables and different types of constraints. After successful validation to standard WDNs, the CE method is applied to a real WDN in India and the results are compared with the existing solutions. It is found that CE method is giving minimum cost solutions (i.e., good quality optimal solutions) in quicker time (i.e., rapid convergence to optimum). Thus, the study concludes that the cross entropy optimization method is an effective optimization method for solving WDN problems, and which can be applied for optimal design of any practical WDN problems.

References

[1] Gupta, I., Gupta, A., and Khanna, P., 1999, "Genetic algorithm for optimization of water distribution systems," Environmental Modelling & software, 24(4), pp.437-446.

[2] Goldberg, D.E., and Kuo, C.H., 1987, "Genetic algorithms in pipeline optimization," Journal of Computing in Civil Engineering, 1(2), pp. 129-141.

[3] Simpson, A. R., Dandy, G. C., and Murphy, L. J.,1994, "Genetic algorithms compared to other techniques for pipe optimization," J. Water Resour. Plang. and Mgmt., ASCE, 120(4), pp. 423-443.

[4] Savic, D.A. and Walters, G.A., 1997, "Genetic algorithms for least-cost design of water distribution networks," J. Water Resour. Plang and Mgmt., ASCE, 123(2), pp. 67-77.

[5] Dandy, G.C., Simpson A.R., and Murphy L.J., 1996, "An improved genetic algorithm for pipe network optimization," Water Resour. Res., 32(2), pp. 449 - 458.
[6] Wu, Z. Y., and Simpson, A. R., 2001, "Competent genetic-evolutionary optimization of water distribution systems," Journal of Computing in Civil Engineering, 15(2), pp. 89-101.
[7] Cunha, M.D.C., and Sousa, J., 1999, "Water distribution network design optimization: simulated annealing approach," J. Water Resour. Plang. and Mgmt., 125(4), pp. 215-221.
[8] Eusuff, M. M., and Lansey, K.E., 2003, "Optimization of water distribution network design using the shuffled frog leaping algorithm," J. Water Resour. Plang. and Mgmt., ASCE, 129(3), pp. 210-225.
[9] Maier, H. R., Simpson, A.R., Zecchin, A. C., Foong, W.K.,Phang, K.Y., Seah, H.Y., and Tan, C.L.,2003, "Ant Colony Optimization for Design of Water Distribution Systems," J. Water Resour. Plang. and Mgmt., ASCE, 129(3), pp. 200-209.
[10] Shibu, A. Reddy, M.J., 2011, "Least cost design of water distribution network by Cross entropy optimization," World Congress on Information and Communication Technologies (WICT), vol.,no., pp..302-306,11-14Dec.2011
doi: 10.1109/WICT.2011.6141262
[11] Rubinstein, R.Y., 1997, "Optimization of computer simulation models with rare events," European Journal of Operations Research, 99, pp. 89-112.
[12] Shannon, C.E., 1948, "A Mathematical theory of communication," Bell System Tech. Journal, 27, pp. 379 - 423.
[13] Kullback, S, and Leibler, R.A., 1951, "On information and sufficiency," Ann. Math. Statics, 22, pp. 79-86.
[14] Fujiwara, O., and Khang, D.B., 1990, "A two-phase decomposition method for optimal design of looped water distribution networks," Water Resour. Res., 26(4), pp. 539-549.
[15] Dijk, M. V**.**, Vuuren, S. V., and Van, Z., 2006, "Optimizing water distribution systems using a weighted penalty in a genetic algorithm," ISSN, Water SA, 34(5), pp. 378 - 478.
[16] Vairavamoorthy, K., and Ali, M., 2000, "Optimal design of water distribution systems using genetic algorithms," Computer-Aided Civil and Infrastructure Engineering, Blackwell, 15(4), pp. 374–382.
[17] Schaake, J. and Lai, D., 1969, "Linear programming and dynamic programming applications to water distribution network design,", Rep. No. 116, Dept. of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

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