# Performance of Modified Iterative Decoding Algorithm for Multilevel Codes in Adaptive OFDM System

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Abstract: In this paper, the Modified Iterative Decoding Algorithm (MIDA) is investigated for decoding Multi-level codes. Adaptive Orthogonal Frequency Division Multiplexing (AOFDM) system is used as system model for this experiment. MIDA is a hard decision decoder that was initially proposed for decoding of Product codes and later for Multi-level codes by same authors. As Multi-level codes are matrix codes we have found that they have structural compatibility with OFDM systems. In OFDM system each subchannel may have different channel state information (CSI) which may be varying over the time. So a Multilevel code with suitable combination of constituent row codes can play a vital role in combating poor channel conditions on OFDM subchannels. A fuzzy rule based system (FRBS) is used for selection of suitable most Multilevel code and modulation symbol. Performance is shown by simulations.

*Keywords:* Modified Iterative Decoding Algorithm, Multi-level Codes, Bit Error Rate, Linear Block Codes.

# I. Introduction

Concatenated codes are mainly categorized into two types. First the serial concatenated codes also called product codes [1] and the second type is parallel concatenated codes also called turbo codes [2]. Product codes were first presented by Elias in 1954 [3]. The structure of Product codes is simple and powerful in which instead of using one long block codes a number of small codes are concatenated that can be decoded in parallel fashion. These are matrix codes having rows encoded by one block code and columns are encoded by another block code.

Multi-level codes belong to the family of Product codes. The only difference is that in Multi-level codes each row may be encoded by a different block code while all the columns are encoded by same block code. This structure is suitable for many adaptive orthogonal frequency division multiplexing (AOFDM) systems, where different code rates may be assigned to different subcarriers based upon their channel state information (CSI). Multi-level codes are also quite practical in a sense that they have been used in many wireless standards nowadays especially in adaptive systems like WIFI (IEEE 802.11n) [4] and WiMAX (IEEE 802.16/e) [5]. Since their structural characteristics are very much compatible with OFDM systems, product codes are recommended for almost all 3<sup>rd</sup> Generation (3G) and 4<sup>th</sup> Generation (4G) systems including wireless local area networks (WLAN) and HYPERLAN standards [6].

In his PhD dissertation, Al-Askary [6] proposed an iterative decoding algorithm for Product codes. That algorithm was based upon List Decoders for rows are columns and designated as the Maximum Likelihood (ML) decoder of product codes [7]. ML decoding is an optimum decoding in which complexity grows exponentially with the codes size and number of iterations.

The modified iterative algorithm (MIDA) was originally proposed for decoding of Product codes by Attaur-Rahman *et al* [8]. It is hard decoding algorithm that significantly reduces complexity of the basic iterative algorithm proposed by [6].

In [9], MIDA was proposed for Multi-level codes and the decoder's performance was investigated over an OFDM system. Moreover, the performance Multi-level codes with different parameters were demonstrated. It was shown that proposed scheme performs significantly better than the best scheme in the same area.

In [10], an adaptive coding and modulation scheme was proposed for OFDM systems in which Product codes were used as forward error correction (FEC) codes and Quadrature Amplitude Modulation (QAM) as modulation scheme. MIDA was used to decode the Product codes in this scheme. A Fuzzy Rule Based System (FRBS) was used to select the suitable code rate and modulation symbol depending upon the channel state information (CSI). It was shown through simulation results that the proposed scheme performs significantly better than HYPERLAN/2 standard scheme.

Similarly, in [11, 12], same Fuzzy Rule Based System is used for various coding schemes like Convolutional codes etc. It was found the fuzzy logic approach performs best in the environments that are vague and unclear and missing certain information.

In [13], a resource allocation and resource leveling technique for heterogeneous SANETs (sensor active networks) environment, is presented. There introduced a RMU (resource management unit) that ensures a cooperative communication and provides features for an on-demand channel relocation.

In [14], Fuzzy Logic is used for suitable web access. In this paper authors presented assessment methodology and model for performance measurement of dynamic websites. It was named as Fuzz-Web; fuzzy logic is used for taking intelligent decision regarding performance measurement.

Rest of the paper is organized as follows: Section 2 presents the basic model; structure and construction of Multi-level Code is given in Section 3; Section 4 is based on Iterative Algorithm. Section 5 presents the proposed Algorithm. Section 6 is consisted of FRBS based adaptive coding and modulation with Multi-level codes, Section 7 covers simulation results while section 8 concludes the paper.

# II. System Model

The system model considered is an OFDM equivalent baseband model with N number of subcarriers [12]. The frequency domain representation of system is given by

$$y_{k} = h_{k} \cdot \sqrt{p_{k}} x_{k} + z_{k}; k = 1, 2, \dots, N$$
 (1)

where  $y_k$ ,  $h_k$ ,  $\sqrt{p_k}$ ,  $x_k$  and  $z_k$  denote received signal, channel coefficient, transmit amplitude, transmit symbol and the Gaussian noise of subcarrier  $k = 1, 2, \dots, N$ , respectively. The overall transmit power of the system is  $P_{total} = \sum_{k=1}^{N} p_k$  and the noise distribution is complex Gaussian with zero mean and unit variance.

It is assumed that the complete channel state information (CSI) at any subcarrier is known to transmitter and receiver including which row code is being used at any subcarrier.

It is also assumed that signal transmitted on the kth subcarrier is propagated over an independent nondispersive single-path Rayleigh Fading channel and where each subcarrier faces a different amount of fading independent of each other. Hence, the channel coefficient of kth subcarrier can be expressed as:

$$h_k = \alpha_k e^{j \theta_k}; k = 1, 2, \dots, N$$
 (2)

where  $\alpha_k$  is Rayleigh distributed random variable of *k*th subcarrier, and the phase  $\theta_k$  is uniformly distributed over  $[0, 2\pi]$ , while *j* is iota symbol since phase is complex.

Fig-1 contains the basic system model used for simulations. In OFDM Systems one big data stream is divided into a number of relatively small data streams by inverse fast Fourier Transform (IFFT). These streams are modulated over orthogonal subcarriers and addition of adequate cyclic prefix makes the system inter-symbol interference (ISI) free.



Figure 1. System Model

## III. Muli-Level Codes

As it is already described that multi-level codes belong to the family of Product codes, so in order to understand the multi-level codes, let's have a look at construction of Product codes. Consider two block codes  $A_1$  and  $A_2$  with parameters  $[n_1, k_1, d_1]$  and  $[n_2, k_2, d_2]$  respectively, where  $n_i, k_i$  and  $d_i; i = 1, 2$  are the length, dimension and minimum Hamming distance ( $d_{\min}$ ) of the code  $A_i$  (i = 1, 2) respectively. Code  $A_1$  will be used as row code while  $A_2$  will be used as column code. The rates of individual codes are  $R_1$  and  $R_2$  respectively given by,

$$R_i = \frac{k_i}{n_i}, i = 1, 2$$
 (3)

The product code  $\Omega$  can be obtained by codes  $A_i$ , i = 1, 2 in the following manner.

- Place k<sub>1</sub>×k<sub>2</sub> information bits in an array of k<sub>2</sub> rows and k<sub>1</sub> columns
- Encode  $k_2$  rows using code  $\mathbf{A}_1$ , which will result in an array of  $k_2 \times n_1$
- Now encode  $n_1$  columns using code  $A_2$ , which will result in  $n_2 \times n_1$  product code.

The resultant product code  $\Omega$  has the parameters  $[n_1n_2,k_1k_2,d_1d_2]$  and the rate will be  $R_1R_2$ . In this way long block codes can be constructed using much shorter constituent block codes.

This concept can also be viewed as that product code  $\Omega$  is intersection of two codes  $\mathbf{A}_1$  and  $\mathbf{A}_2$ . Where  $\mathbf{A}_1$  is a code represented by all  $n_2 \times n_1$  matrices whose each row is a member of code  $\mathbf{A}_1$ , similarly  $\mathbf{A}_2$  is a code represented by all  $n_2 \times n_1$  matrices who's each column is a member of code  $\mathbf{A}_2$ . This can be written as;

$$\mathbf{\Omega} = \mathbf{A}_{1}^{'} \cap \mathbf{A}_{2}^{'} \tag{4}$$



Figure 2. Structure of the Product code

Now as far as multi-level codes are concerned the rows of these codes are encoded by a set of block codes having same dimensions but different code rates while column is mostly encoded by one block code.

Consider a set of linear block codes; that is  $C = [A_q]; 1 \le q \le S$ , where S is cardinality of the set.  $A_q = [n, k_q, d_q]$  are the elements of the set, where  $k_q$  and  $d_q; q = 1, 2..., S$  are the dimension and minimum Hamming distance  $d_{\min}$  of the code respectively and n represents code length. The dimensions of all codes taken must be same in order to make the multi-level code a matrix code. The rates of individual row codes are given by,

$$R_q = \frac{k_q}{n}, q = 1, 2, \dots S$$
 (5)

The column code  $B = [n_c, k_c, d_c]$  where  $n_c, k_c$  and  $d_c$  represents dimension, size and minimum distance of the column code. The multilevel code  $\Theta$  can be defined as set of all matrices whose rows belong to set C and whose columns belong to code B.

## IV. Iterative Algorithm

The idea of Iterative decoding algorithm for Product code was originally proposed by Al-Askary [6], for reference purpose it is restated here. The decoder is consisted of two sub-decoders, namely one for rows  $\phi$  while other for columns  $\psi$ . The received  $n_2 \times n_1$  matrix **R** can be written as;

$$\mathbf{R} = \mathbf{X} + \mathbf{N} \tag{6}$$

where **X** and **N** are transmitted and noise matrices of dimensions  $n_2 \times n_1$  respectively. The received matrix is fed to row and column decoders in succession at each stage of iterative algorithm. At first row decoder gives its suggested solution, which is further fed to column decoder, which after processing returns its suggested solution, this process continues in all stages of decoder until a stopping criterion met. A detail of these sub-decoders is given in turn. Fig-3 shows the flowchart of iterative decoding algorithm and fig-4 shows the status of *i*th state of the iterative decoder.

#### A. Row Decoder $\phi$

This decoder receives a matrix and as a result provides another matrix as a solution in which rows of the incoming matrix are corrected. At the *i*th stage of iterative decoder this sub-decoder takes the previous  $n_2 \times n_1$  solution  $\mathbf{S}^{i-1}$  as input and creates a list  $\mathbf{L}_1^i$  that consisted of  $n_2$  sub-lists, where each sub-list is maintained for the corresponding row in  $\mathbf{S}^{i-1}$ . As we have already mentioned that we have to knowledge that which row is encoded by which constituent code from set C.

Each sub-list contains those code words in code space of  $A_q$  whose distance from that row is less than or equal to  $e_{A_q}$ , in ascending order, where  $e_{A_q}$  is referred as decoding radius of row decoder corresponding to block code q in set C. The list  $\mathbf{L}_1$  at decoder's stage i can be represented as the Cartesian product of all sub-lists, namely;

$$\mathbf{L}_{1}^{i} = \prod_{j=1}^{n_{2}} \xi_{e_{\mathbf{A}q}}^{j} \left( \mathbf{S}_{j,:}^{(i-1)}, \mathbf{A}_{q} \right)$$
(7)

Where  $\xi_{e_{Aq}}^{j}$  is a sub-list that contains the candidates for the *jth* row in matrix **S**<sup>*i*-1</sup>. After the list is prepared, decision will be taken as described below.

## B. Decision criteria for row decoder $\phi$

Row decoder  $\phi$  returns its suggested solution  $\mathbf{T}^{i}$ , in the following way;

.....

$$\mathbf{T}^{i} = \arg \underbrace{\mathbf{t} \in \mathbf{L}_{1}^{i}}_{D(\mathbf{t},\mathbf{R}) > \min(\alpha^{i-1},\beta^{i-1})} D(\mathbf{t},\mathbf{R}) \quad (8)$$

where  $D, \alpha, \beta$  are defined as;

- D = some distance defined like Hamming distance
- α<sup>i</sup> = D(T<sup>i</sup>, R); distance of *i*th stage row solution and received matrix
- $\beta^i = D(\mathbf{S}^i, \mathbf{R})$ ; distance of *i*th stage column solution and received matrix

So in other words the row decoder at stage *i* chooses the member of list  $\mathbf{L}_{1}^{i}$  that is closest to **R**, but at a distance greater than the solutions suggested in previous stages, i.e. , min( $\alpha^{i} - 1$ ,  $\beta^{i} - 1$ ). Then this suggested solution will be further processed by column decoder.

#### C. Column Decoder $\psi$

Similar to that of row decoder column decoder concerns with the columns of received matrix  $\mathbf{R}$ . At *ith* stage of iterative decoder this sub-decoder takes the previous  $n_2 \times n_1$  solution  $\mathbf{T}^i$  that was suggested by *i*th stage row decoder, as input and creates a list  $\mathbf{L}_2^i$  that consisted of  $n_1$  sub-lists, where each sub-list is populated for the corresponding column in  $\mathbf{T}^i$ . Each sub-list contains those code words in *B* whose distance from that column is less than or equal to  $e_B$  in ascending order, where  $e_B$  is referred as decoding radius of column decoder. The list  $L_2$  at decoder's stage *i* can be represented as the Cartesian product of all sub-lists;

$$\mathbf{L}_{2}^{i} = \prod_{j=1}^{n_{1}} \xi_{e_{B}}^{j} \left( \mathbf{T}_{:,j}^{(i)}, B \right)$$
(9)

where  $\xi_{e_B}^{j}$  is a sub-list that contains the candidates for the *jth* column in matrix **T**<sup>*i*</sup>. After the list preparation, decision will be taken in the following manner.

## D. Decision criteria for column decoder $\psi$

Column decoder  $\psi$  returns its suggested solution  $S^i$ , in the following way

$$\mathbf{S}^{i} = \arg \underbrace{\mathbf{s} \in \mathbf{L}_{2}^{i}}_{D(\mathbf{s}, \mathbf{R}) > \min(\alpha^{i}, \beta^{i-1})} D(\mathbf{s}, \mathbf{R})$$
(10)

Where  $D, \alpha, \beta$  are same as defined above.

So in other words the column decoder at stage *i* chooses the member of list  $\mathbf{L}_2^i$  that is closest to  $\mathbf{R}$ , but at a distance greater than the solutions suggested in previous stages, than is,  $\min(\alpha^i, \beta^{i-1})$ .

## E. Stopping Criteria

This row/column decoding at each stage will go on in turn till the number of stages that are adjusted by the user. Then the last stage solution will be the ultimate decoding solution.



Figure 3. Flow chart of iterative decoder



Figure 4. *i*th stage of iterative decoder

#### V. Proposed MIDA for Mult-level Codes

Proposed Modified Iterative Decoding Algorithm is a revised version of Iterative decoding algorithm proposed by [6]. It is a hard decision decoder. Syndrome decoding of linear block codes is used for complexity reduction [9]. In this way number of rows/columns, for which lists are to be built, is reduced significantly. Main changes in previous Iterative decoding algorithm are:

• In row decoder rows will be firstly passed through a Syndrome check. If the Syndrome of any row results in 0 (i.e. row is correct), then no sub-list will be populated for that row and the row itself will be returned as a decoded solution. Mathematically,

$$\mathbf{S}_{i:} \cdot \mathbf{H}_{a}^{T} = \mathbf{0} \qquad (11)$$

where  $\mathbf{H}_q$  is parity check matrix of the corresponding row code  $\mathbf{A}_a$ ; q = 1, 2, ..., S

• In column decoder each column will be checked by Syndrome decoder. If the Syndrome of any column results in **0** (i.e. column is correct), then no sub-list will be populated for that column and the column will be return as a decoded solution. Mathematically,

$$\mathbf{T}_{a,j} \cdot \mathbf{H}_B^T = \mathbf{0} \qquad (12)$$

where  $\mathbf{H}_{B}$  is parity check matrix of the column code B

- Sub-lists  $\xi_{e_{Aq}}, \xi_{e_B}$  will be generated only for those rows and columns respectively, who's Syndrome wouldn't result in **0**.
- Decoding radii of rows decoder will be chosen as (t<sub>Aq</sub> +1); where t<sub>Aq</sub> is error correction capability of row code A<sub>q</sub>;q = 1,2,3,...,S in set C.
- Decoding radius of column decoder will be chosen as  $(t_B + 1)$ ; where  $t_B$  is error correction capability of column code *B*

So the Equ-7 and Equ-9 will be changed to two new Equ-13 and Equ-14 which help great reduction in decoding complexity for the same state of art.

$$\tilde{\mathbf{L}}_{1}^{i} = \prod_{j=1}^{n_{2}} \xi_{e_{A_{q}}}^{j} (\mathbf{S}_{j,:}^{(i-1)}, \mathbf{A}_{q}); n_{2}^{'} < n_{2}$$
(13)  
$$\tilde{\mathbf{L}}_{2}^{i} = \prod_{j=1}^{n_{1}^{'}} \xi_{e_{B}}^{j} (\mathbf{T}_{:,j}^{(i)}, \mathbf{B}); n_{1}^{'} < n_{1}$$
(14)

Similarly the Equ-8 and Equ-10 will be converted to;

$$\widetilde{\mathbf{T}}^{i} = \arg \underbrace{\operatorname{arg}}_{D(\mathbf{t},\mathbf{R}) > \min(\alpha^{i-1},\beta^{i-1})}^{\min} D(\mathbf{t},\mathbf{R}) \qquad (15)$$

$$\tilde{\mathbf{S}}^{'} = \arg \underbrace{\mathbf{s}}_{D(\mathbf{s},\mathbf{R}) > \min(\alpha^{i},\beta^{i-1})} D(\mathbf{s},\mathbf{R})$$
(16)

# Proposed MIDA Algorithm for Multilevel Codes Let $\mathbf{R}_{n_2 x n_1}$ be the received code matrix

1. while ( $i \ll$  Max no of iterations) do

- a) If ((each row of **R** is the member in the corresponding row code in set C) and (each column of **R** is the member in column code *B*)) then go to step b, otherwise to step c
- b) Return **R** as the decoded solution and go to step 2
- c) Mark those rows in **R** that are members in the corresponding row code in set C and those columns that are member of column code (using Syndrome check), respectively
- d) Make lists for unmarked rows  $(n_2)$  and columns

 $(n_1)$  using Equ-13 and Equ-14, with decoding

radii  $(t_{A_a} + 1)$  and  $(t_B + 1)$  respectively

- e) Take decisions for suggested solution in row/column decoders at *i*th stage of iteration using Equ-15 and Equ-16 respectively.
- f)  $\mathbf{R} = \mathbf{S}^i$
- g) i = i + 1 go to 2
- 2. Exit

#### VI. Adaptive Coding and Modulation using FRBS

Adaptive coding and modulation for OFDM system using Fuzzy Rule Base System was originally proposed by [10]. We are utilizing the same technique here for adapting Multi-level code rate and modulation scheme with respect to the changing channel state environment at OFDM subcarriers. The adaptation mechanism is shown in fig-5. Similar work is done in [11] and [12] for convolutional codes. FRBS was used to choose the optimum modulation code pair for the given channel state information at individual subchannels of OFDM system after each transmission interval.



Figure 5. Proposed Adaptation Model

Signal is transmitted through the OFDM physical (PHY) layer (air interface). After passing through the channel, PHY layer receiver obtains the channel estimates. Along with the channel estimates, quality of service demand is fed to the link adaptation block (LAB). LAB which is actually Fuzzy Rule Based System (FRBS) suggests the optimum modulation code pair that maximizes the OFDM system throughput while satisfying certain constraints.

#### A. Coding Scheme

Coding schemes used for this framework are set of Multi-level codes. The set of row codes and column codes used in this paper are listed in table1. All of these codes are BCH codes.

TABLE I. CODING PARAMETER

Sr	Row Code	Column Code	Product Code	Code rate
C1	[63,63,1]	[63,57,3]	[3969,3591,3]	0.9
C2	[63,57,3]	[63,57,3]	[3969,3249,9]	0.82
C3	[63,51,5]	[63,57,3]	[3969,2907,15]	0.73
C4	[63,36,11]	[63,57,3]	[3969,2052,33]	0.51
C5	[63,63,1]	[63,63,1]	[3969,3969,1]	1
C6	[63,57,3]	[63,63,1]	[3969,3591,3]	0.9
C7	[63,51,5]	[63,63,1]	[3969,3213,5]	0.8
C8	[63,36,11]	[63,63,1]	[3969,2268,11]	0.57

So set of code is consisted of four different product codes. That is

$$C = \{C_i\}; 1 \le i \le 8 \tag{17}$$

The reasons for selection of these codes are as under.

- All codes are of same length would be helpful in hardware implementation
- Same length of row codes make it possible for decoding since if we use different length codes then upon receiving received matrix may not be formulated

#### B. Modulation Scheme

The modulation scheme used for this experiment is Quadrature Amplitude Modulation (QAM) which is recommended by many OFDM standards. Following set of modulation schemes is used. That is

$$M = \{2, 4, 8, 16, 32, 64, 128\}$$
(18)

So with these coding and modulation sets we have twentyeight possible modulation code pairs (MCP) by a Cartesian product of the sets C and M. This can be given by the expression.

 $P = C \mathbf{x} M = \{ (c_i, m_i); \forall c_i \in C, \forall m_i \in M \}$ (19)

After deciding modulation and coding schemes for this framework, all of the possible combinations of modulation code pairs are plotted in subsequent figures. In fig-6, all modulation schemes namely from 2QAM to 128QAM are plotted using Product Code C5 as listed in table 1. Similarly, in fig-7 and fig-8 different QAM modulations are plotted using Product codes C2 and C3 respectively.



Figure 6. Performance of different QAM schemes using C5 as row code



Figure 8. Performance of different QAM schemes using C3

We have used a fuzzy rule base system (FRBS), which is capable of deciding the best modulation code pair (MCP) for the next transmission, based upon the heuristics. Fuzzy logic is best suited for the situations that are vague, ambiguous, noisy or missing certain information. There are many ways we o build a Fuzzy Rule Base System, we have used *table lookup scheme for this purpose*. The lookup table is given in fig-9. This table shows the facts extracted for simulated performance of different codes and modulation pairs in previous section. It can be stated as "for a given received SNR and a fixed QoS, which MCP maximizes the throughput". Received signal to noise ratio is expressed in level 1 to level 9 and Quality of Service are given like poor, med, good and high that is  $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$ ,  $10^{-4}$  respectively.

			QoS>>		
		Poor	Med	Good	High
	L9	P9	P9	P9	P25
	L8	P8	P17	P16	P24
	L7	P7	P16	P15	P23
	L6	P6	P15	P21	P14
SN	L5	P5	P14	P20	P22
R-	L4	P4	P13	P13	P2
٨	L3	P3	P12	P2	P11
	L2	P2	P11	P19	P10
	L1	P1	P10	P18	P18

Figure 9. Lookup Table for FRBS Creation

## C. Rate Optimization

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In order to maximize the rate for OFDM system following constrained optimization problem is considered.

$$\max \quad R_{Total} = \frac{1}{N} \sum_{k=1}^{N} r_k$$
  
s.t,  
$$BER_{Total} \leq BER_T \quad and \qquad (20)$$
$$P_{Total} = \sum_{i=1}^{N} p_i < P_T$$

where  $r_k = (\log_2(M))_k R_k$  is the product of code rate and modulation bits/symbol over *k*th subcarrier.  $P_T$  is the available transmit power.  $BER_T$  is target BER that depends upon a specific quality of service (QoS) request or application requirement. The possible QoS assumed are  $BER_T = 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}$  while N is total number of subcarriers in OFDM system. The above cost function is optimized by the proposed Fuzzy Rule Base System.

It will be decided that which modulation code pair is suitable for transmission based upon the average channel state information (CSI) at the subcarriers and the Quality of Service demand. We have used the table look-up scheme for design of this fuzzy rule base system using the following steps. The input-output pairs needed for design of FRBS are provided in figure 8. They are of the form;

$$(x_1^p, x_2^p; y^p); p = 1, 2, 3.....M$$
 (21)

{IF  $(x_1 \text{ is Good and } x_2 \text{ is L7})$  THEN y is P15}

Following is the brief description of different components of fuzzy rule based system used. Design of the FRBS is carried out in MATLAB 7.0 standard Fuzzy System Toolbox. The interface of the toolbox is given in fig-10 and fig-11.

# G. 🛃 Rule Editor: Ultimate 1.001 File Edit View Options If (SMR is L0) and (MLBER is C0) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C3) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C3) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C3) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C3) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C3) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C3) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C6) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C6) than (MCPair is P2) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) (1) If (SMR is L0) and (MLBER is C0) than (MCPair is P1) If (SMR is MCPair is P1) (1) If (SMR is MCPair is P1) (1) If (SMR is MCPair is P1) If (SNR is L0) and (MLBER is Q10) then (MCPair is P1) (1 11. If (SNR is L0) and (MLBER is Q11) then (MCPair is P1) (1) and M. DER .... MCPar in COURT IN 04 Viewh Cons 0/ 8/16 Delete rule Add rule Change rule 1 FIT N res illiants Close



Figure 11. Fuzzy Rule Base System at a glance

## D. Fuzzy Sets

Sufficient numbers of fuzzy sets are used to cover the input output spaces. There are two input variables average received SNR and QoS that represents a BER. There is one output variable for modulation code pair MCP. All of these input and output variables are depicted in fig-12, fig-13 and fig-14 respectively.

There are nine, four and eighteen fuzzy sets used for the variables SNR, QoS and MCP, respectively, where SNR and QoS are input variables while MCP is output variable.

## E. Fuzzifier

Standard triangular fuzzifier is used with AND as MIN and OR as MAX.

## F. Rule Base

Rule base contains rules against all the IO pairs. As there are nine sets (L1 to L9) for first input variable named SNR and about four sets (low, medium, good and high) for input variable QoS. Hence there are 36 rules in rule base.

# Inference Engine

Standard Mamdani Inference Engine (MIE) is used that will infer which input pair will be mapped on to which output point.

## De-Fuzzifier

Standard Center Average Defuzzifier (CAD) is used for defuzzification. This is because it fulfills all the requirements of a good de-fuzzifier. Like it is computationally light and its performance is better than its peers.









Figure 15. Rule surface

Fig-15 shows the rule surface that shows that by increasing SNR the throughput is maximized. Also on the other hand for poor QoS throughput is more than that of high QoS. A combined effect of both input variables namely SNR and QoS can be seen in that figure. For the highest value of SNR and lowest value of QoS, throughput of the system approaches to 5bits/s/Hz.

# VII. Simulation Results

The components of multilevel codes used in the simulation are given in the table-I, while simulation parameters are given in table-II. There are four row codes with different code rates but same code length because according to structure ultimately it should become a matrix.

Two different column codes are investigated in first case code rate is one that is no redundancy is introduced while in second case there is redundancy overhead of six bits. Then obviously the multi-level codes with [63, 57, 3] block code performs better than that of multi-level codes with [63, 63, 1] block code.

The scheme is tested for a range of signal to noise ratio (SNR) and bit error rate (BER) is calculated which is demonstrated in fig-16. The simulation parameters are chosen same as that were in [6] so that proposed decoder's performance can be highlighted.

Simulation results show vitality of proposed algorithm over the basic algorithm with a reduced complexity. The comparison is given between conventional iterative algorithm and proposed modified iterative decoding algorithm for multi-level codes with and without column redundancy respectively. If we introduce column redundancy then it means we have to scarify some subchannels for carrying the redundant information. In this way we have to compromise the throughput but a better bit error rate (quality of service) is guaranteed. Similarly, if there is no column redundancy then code rate will not be compromised but bit error rate may be more.

TABLE II. SIMULATION PARAMETER

Sr.	Parameter name	Value
1	Coding Schemes	Multilevel Code
2	Code rates	1, 0.9, 0.8, 0.57
2	Modulation	2, 4, 8, 16, 32,
3	Schemes	64, 128 QAM
4	Bits/symbols in	1, 2, 3, 4, 5, 6, 7

Sr.	Parameter name	Value
	modulation	
5	Total MCPs	4x7=28
5	OFDM Standard used	HYPERLAN/2
6	Number of subchannel	63
7	Minimum	0.57x1=0.57bits/
'	throughput MCP	s/Hz
8	Maximum throughput MCP	1x7=7bits/s/Hz
9	Adaptation	Both modulation and code
10	Adaptation Criteria	Fuzzy Rule Base System

Moreover, as it is already told that we have assumed that complete channel state information is available at both transmitter and receiver. So the information that which row is encoded by which component code is already available to the decoder. So as code matrix received list of appropriate code will be populated according to the procedure described in previous sections.

Similarly, it can be seen that the native decoding algorithm demands almost 5dB more in terms of signal to noise ratio for possessing the same performance. If we observe the bit error rate at 5dB signal to noise ratio then a significant difference is notable. That is more than two order difference and proposed scheme gives a 5dB gain over the previous work. Hence proposed scheme outperforms compare to native scheme.

In fig-17, proposed scheme is compared for various quality of service (QoS) like average BER=10e-1, 10e-2, 10e-3 and 10e-4. In this way QoS was fixed initially then depending upon the received signal to noise ratio (SNR), most appropriate modulation code pair (MCP) was chosen using Fuzzy Rule Base System (FRBS), for entire OFDM system, then the product of modulation rate and code rate so called modulation-code-product is considered as throughput is plotted.

In fig-18, proposed scheme is compared with the Adaptive Coding scheme proposed by Al-Askary in this PhD dissertation [6], where HYPERLAN/2 standard was compared, the adaptation criteria was based upon SNR thresholds. As simulation results reveal, proposed scheme profoundly performs better than that of proposed by Al-Askary as well as HYPERLAN/2 standard.

In first graph, it is revealed that the performance with the Multilevel codes having zero column code redundancy ends up in a high code rate that is 100Mbps at 30dB SNR. But obviously in this way Quality of Service may be little compromised.

In second graph of fig-18 codes with column code having redundancy would cause little degradation in system throughput that it went down to 80Mbps because in this case we have to scarify subcarriers for redundancy but with an improved QoS compared to previous case. Both cases outperform compare to scheme proposed by Al-Askary [6] and HYPERLAN/2 standard.



Figure 16. Performance comprarison of proposed algorithm



Figure 17. Comparison of proposed scheme for various QoS in a HYPERLAN/2 environment



Figure 18. Comparison of proposed scheme with different schemes

## in redundancy VIII. Conclusions

1.

Native decoder for Multi-level codes with column reductance. Native decoder for Multi-level codes with column reductance of Multi-level codes is investigated for an Adaptive Orthogonal Frequency Division Multiplexing (AOFDM) environment. In which, a Fuzzy Rule Based System is employed for adapting the transmission parameters.

> MIDA is a suboptimum iterative decoding algorithm that reduces the complexity of its native counterpart in which List Decoding is employed. By using the concept of Syndrome Decoding, MIDA significantly reduces the search space. It is also noted that the performance of MIDA is as good as original iterative decoding algorithm for Multilevel codes.

> Proposed scheme was compared with OFDM HYPERLAN/2 standard as well as with a similar work namely Adaptive Coding for OFDM System by Al-Askary [6] and significance of proposed scheme is shown by using simulation results. Significance of proposed scheme is due to the following factors,

Wide range of constituent row codes for Multilevel codes.

A relatively low complexity decoder for Multilevel codes.

Every subcarrier may be assigned a different code rate and different modulation scheme depending upon CSI

Wide range of modulation code pairs to handle almost all possible channel conditions.

A Fuzzy Rule Base System to choose suitable most combination of code and modulation scheme based upon a specific Quality of Service and average received channel power to interference noise ratio (CINR).

A constrained optimization problem is focused, that is solved by employing Multilevel codes with MIDA decoder under supervision of FRBS.

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