Wavelet based Noise dressing for reliable Cardinality Constrained Financial Portfolio Sets

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Agenda

- Problem of Financial Portfolio selection / optimization
- Cardinality constrained Portfolio selection problem
- Historical Estimates
- Literature Survey on various filters
- Wavelet based filter
- Hybridization of Wavelet based filter and K-means cluster analysis
- Performance analysis
- Conclusion

Portfolio Management

- A mathematical modeling with deep economic understanding of the market is required for real investment decision.

- Hence portfolio theory is a more versatile tool in the modern investment.

- It helps in estimating risk of an investor's portfolio, with lack of information, uncertainty and incomplete knowledge of reality.

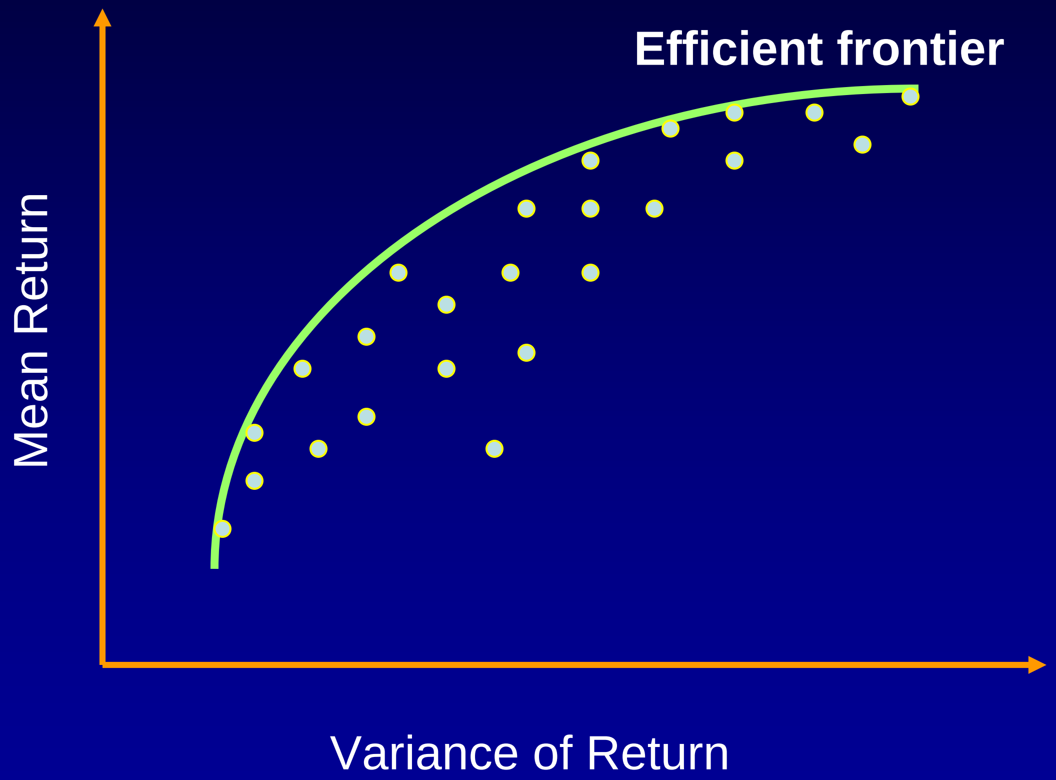
Problem of Portfolio Selection / Optimization

- Distribution of a capital over different investment options such as stocks, bonds and securities is called a portfolio.

- Portfolio optimization deals with the problem of determining the optimal proportion of capital that the investor should put on each line of his portfolio to meet the twin objective of maximizing expected return and minimizing risk.

- The solution is achieved by graphing a risk-return tradeoff curve called efficient frontier, giving the maximum return for a given level of risk or, alternatively, the minimum risk for an expected return.

Problem of Portfolio Selection / Optimization



Problem of Portfolio Selection / Optimization

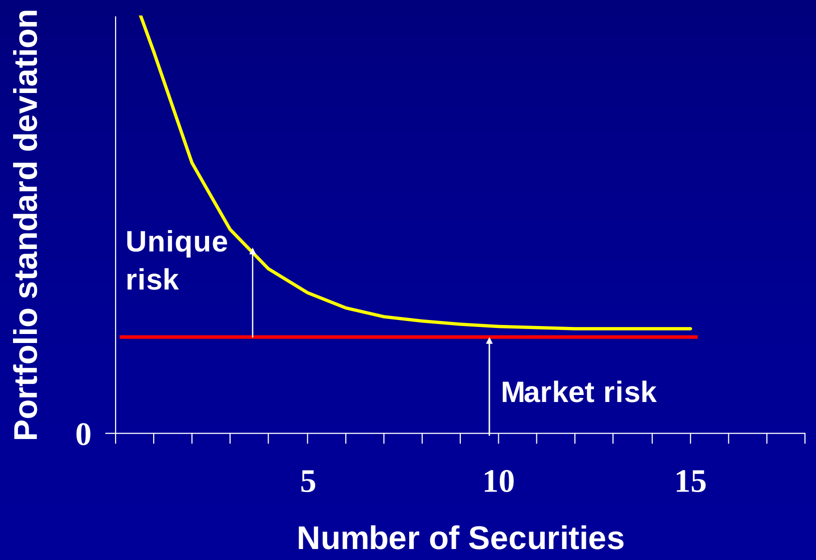
Individual stock return may not be normally distributed, but a portfolio consists of more and more stocks would have its return increasingly close to being normally distributed.

1st moment: Mean = Expected return of portfolio

2nd moment: Variance = Variance of the return of portfolio (RISKNESS)

Unique Risk - Risk factors affecting only that firm. Also called "diversifiable risk."

Market Risk - Economy-wide sources of risk that affect the overall stock market. Also called "systematic risk."



Mathematical Formulation

Objective function

Minimize (risk)

Maximize(return)

Objective function

Minimize $$\sum_{i=1}^{N}\sum_{j=1}^{N} p_i . c_{ij} . p_j$$

Maximize $$\sum_{i=1}^{N} \mu_i . p_i$$

Subject to $$\sum_{i=1}^{N} p_i = 1$$

$$0 \le p_i \le 1$$

Unconstrained Portfolio Selection Problem

The unconstrained weighted portfolio selection problem is given as

Minimize

$$\lambda\left(\sum_{i=1}^{N}\sum_{j=1}^{N} p_i . c_{ij} . p_j\right) + (1-\lambda)\left(-\sum_{i=1}^{N} p_i . \mu_i\right)$$

Subject to

$$\sum_{i=1}^{N} p_i = 1$$

$$0 \le p_i \le 1 \qquad \text{i=1,2….N}$$

Cardinality Constrained Portfolio selection problem

The cardinality constrained weighted portfolio selection problem is

Minimize

$$\lambda\left(\sum_{i=1}^{N}\sum_{j=1}^{N} p_i . c_{ij} . p_j\right) + (1-\lambda)\left(-\sum_{i=1}^{N} p_i . \mu_i\right)$$

Subject to

$$\sum_{i=1}^{N} p_i = 1$$ **Basic Constraint**

$$0 \le p_i \le 1$$ i=1,2….N

$$\sum_{i=1}^{N} x_i <= K$$ **Cardinality Constraint**

x_i = 0 or 1, i = 1, 2…N

Historical Estimates

- Markowitz [1952] laid a quantitative framework using Mean-Variance model termed as Modern Portfolio Theory.
- The key inputs are expected mean and correlation (Covariance) matrices obtained from historical return series.
- Covariance matrix determined from empirical financial return series contains a high amount of noise.
- For practical reasons, the finite length of time series inevitably leads to noise (Measurement error)

Effects of Estimation Error

- Affects the estimation of future risks on optimized portfolios.
- Causes uncertainty in portfolio estimation.
- Leads to serious instability and a high degree of degeneracy in optimal portfolios.
- Makes the entire applicability of the theory more dangerous and questionable.

Literature Survey

- Galluccio, 1998; Laloux Laurant et al., 1999 and Plerou et al., 1999 suggested that presence of high degree of noise in covariance matrix constitutes an intriguing paradox.
- Further more, Laloux Laurent et al.,1999 concluded that "Markowitz portfolio optimization scheme based on a purely historical determination of the correlation matrix is inadequate".

Hence , there is a need to devise the estimation models for removing noise from the correlation matrices.

Literature Survey

- Jorion 1986; Frost and Sararino, 1986; Chopra et al., 1993 and Ledoif and Wolf, 2003 have used pure statistical estimation methods such as Principal Component Analysis or Bayesian Shrinkage estimation.

- Papp et al., 2005; Snarska Malgorzata and Jakub Krzych, 2006 and Laloux Laurent et al., 2007 have studied RMT filtered covariance matrices for portfolio optimization.

- Mantegna, 1999; Giada and Marsili, 2001; Maslov, 2001; Bernaschi et al., 2002; Kullmann et al., 2002; Bonanno et al.,2004 and Basalto et al., 2005 have also proposed other filtering procedures based on clustering in the econophysics literature.

Literature Survey

- Following them, Tola et al., [2005] demonstrated their claims only for the single linkage and average linkage clustering algorithms and also proved that clustering algorithms acted as better reliable filters when compared to RMT based filtering methods.

- Later on Pai and Michel [2007, 2008] studied the application of k-means clustering for portfolio optimization and concluded that k-means cluster analysis outperforms Markowitz and RMT filtered models.

- Meyer [1993] has studied the application of wavelets pertaining to denoising of data, in a wide spectrum of disciplines.

Wavelet Shrinkage denoising methods

- Wavelet shrinkage denoising methods are non-parametric and non-linear methods.
- This method has made significant inroads in almost many applications.
- Hence, we experiment this denoising method for financial covariance matrix.
- Wavelet denoising guarantees the reduction of high frequency noises.
- Symlet function has been chosen as the wavelet function for denoising financial covariance matrix, because of its higher correlation with the empirical covariance matrix.

Wavelet Coefficient Shrinkage technique for denoising

Let the noisy covariance matrix is given as,

$$S_{ij}(t) = C_{ij}(t) + \varepsilon_{ij}(t), \quad i = 1,2 \ldots N, \; j = 1,2 \ldots N$$

Step 1: **Calculate the wavelet coefficient matrix**

$$\omega = \psi(S) = \psi(C) + \psi(\varepsilon)$$

Step 2: **Modify the detail coefficients**

$$\omega \quad \rightarrow \quad \hat{\omega}$$

Step 3: **Inverse the modified detail coefficients**

$$\hat{C} = \psi^{-1}(\hat{\omega})$$

Hybridizing K-means cluster analysis and Wavelet filters for better reliability

- **First, the empirical covariance matrix obtained from historical return series is denoised using wavelet based filter.**

- **Then the denoised covariance matrix and the expected return are used as input to k-means cluster analysis to group N assets into K clusters.**

- **Now, from each cluster, an asset is selected randomly or preferably to satisfy cardinality constraint.**

Performance Analysis

- Several analysis were carried out on Bombay Stock Exchange (BSE 200 index: July 2001 to July 2006) and Tokyo Stock Exchange (Nikkei 225 index: March 2002 to March 2007) data sets.

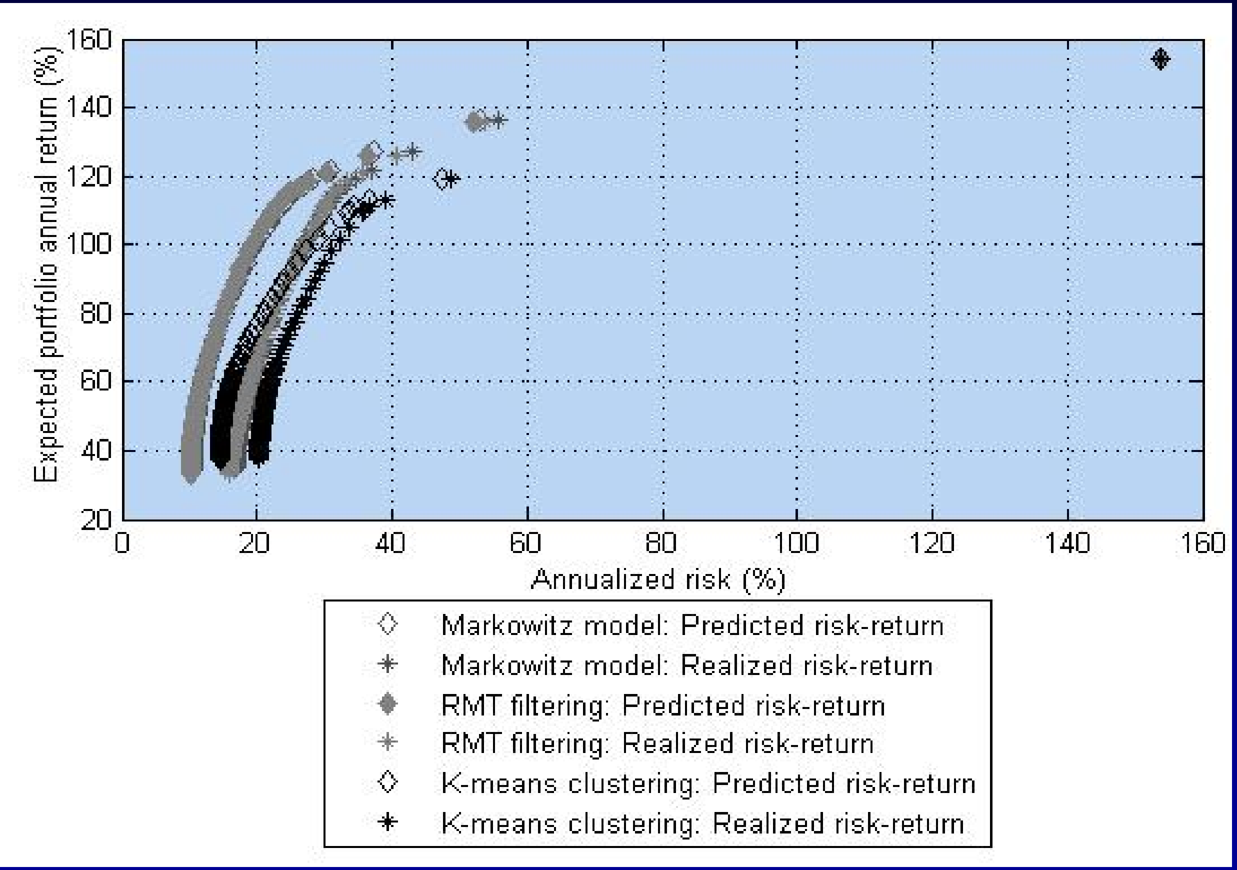


Figure 1(a). Predicted and Realized risk-return graphs for the Markowitz model, RMT filter based model and Investable universe (K=20) for BSE200 (July 2001 – July 2006) data set.

Performance Analysis

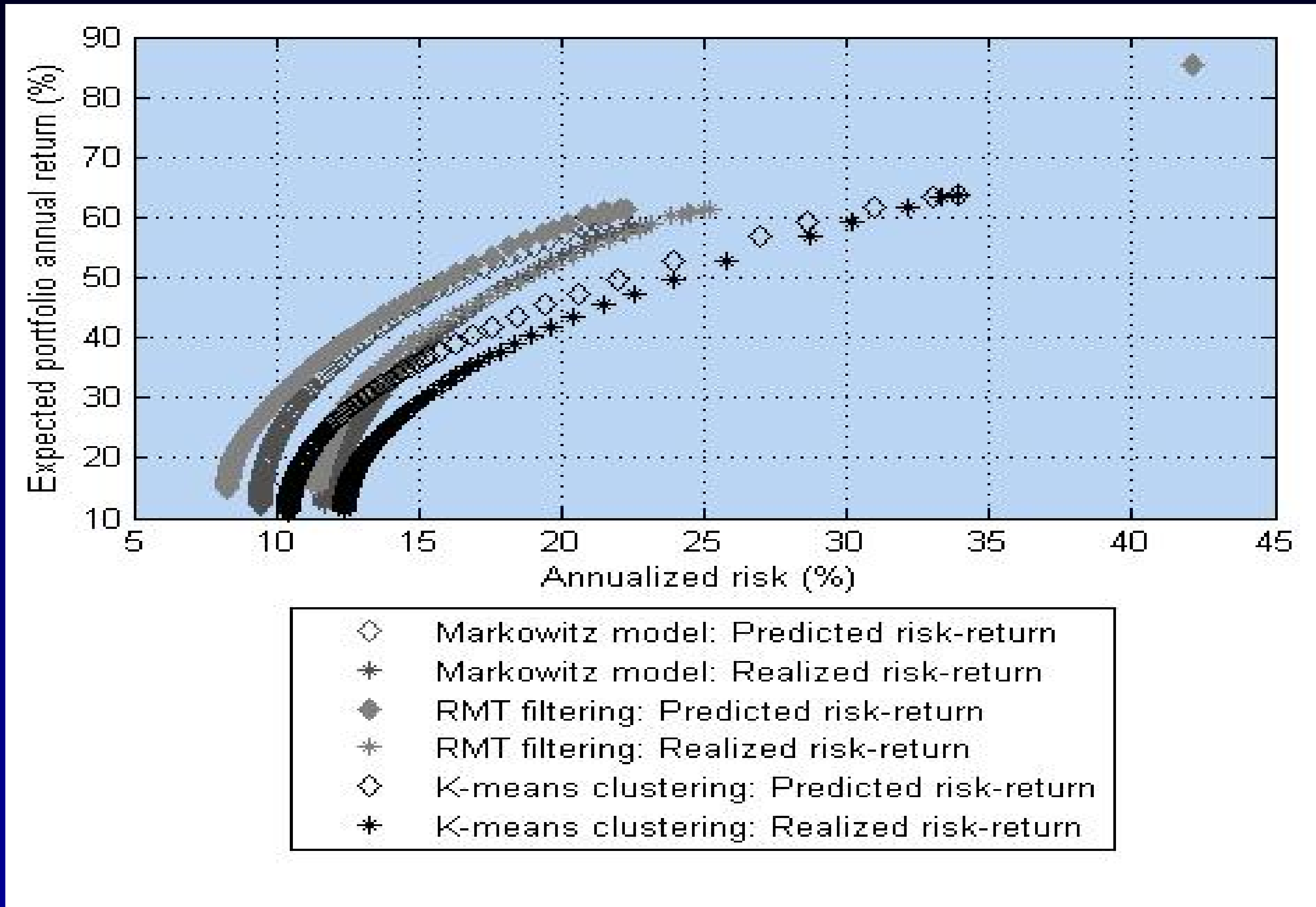


Figure 1(b). Predicted and Realized risk-return graphs for the Markowitz model, RMT filter based model and Investable universe (K=50) for Nikkei225 (March 2002 – March 2007) data set.

Performance Analysis

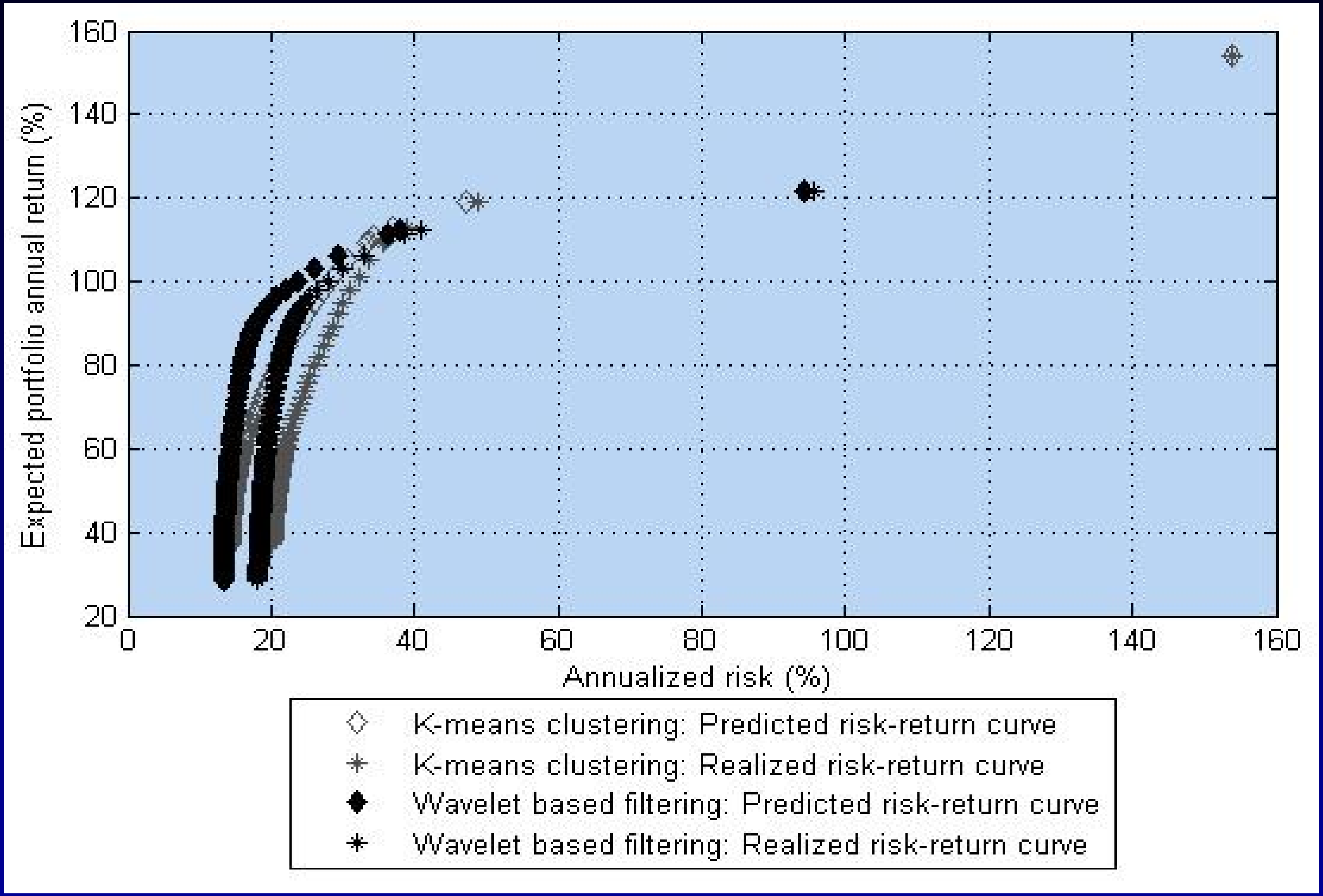


Figure 2(a). Predicted and Realized risk-return graphs for the k-means clustered based model (K=20) and Wavelet based filter model for BSE200 (July 2001 – July 2006) data set.

Performance Analysis

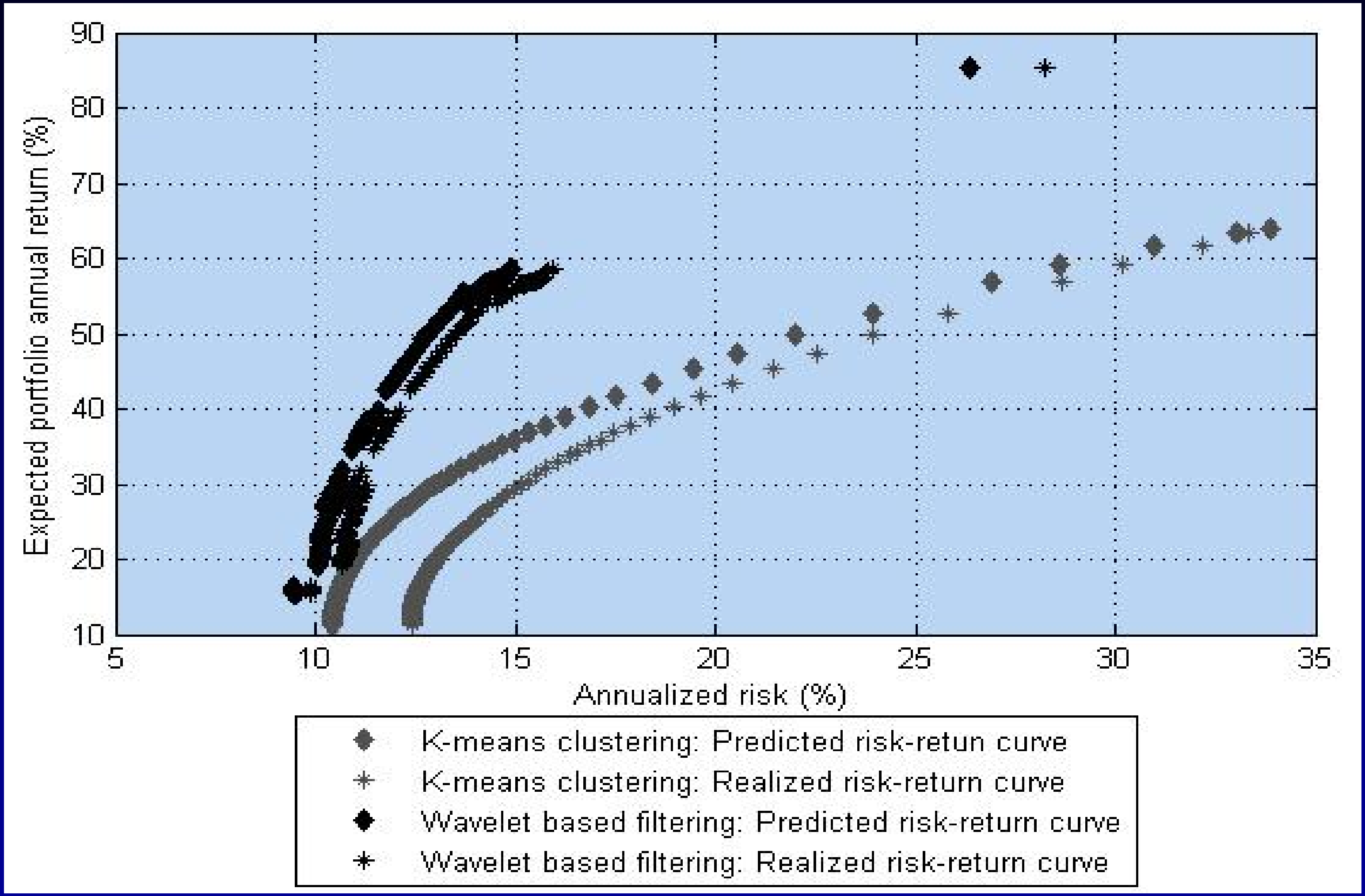


Figure 2(b). Predicted and Realized risk-return graphs for the k-means clustered based model (K=50) and Wavelet based filter model for Nikkei225 (March 2002 – March 2007) data set.

Performance Analysis

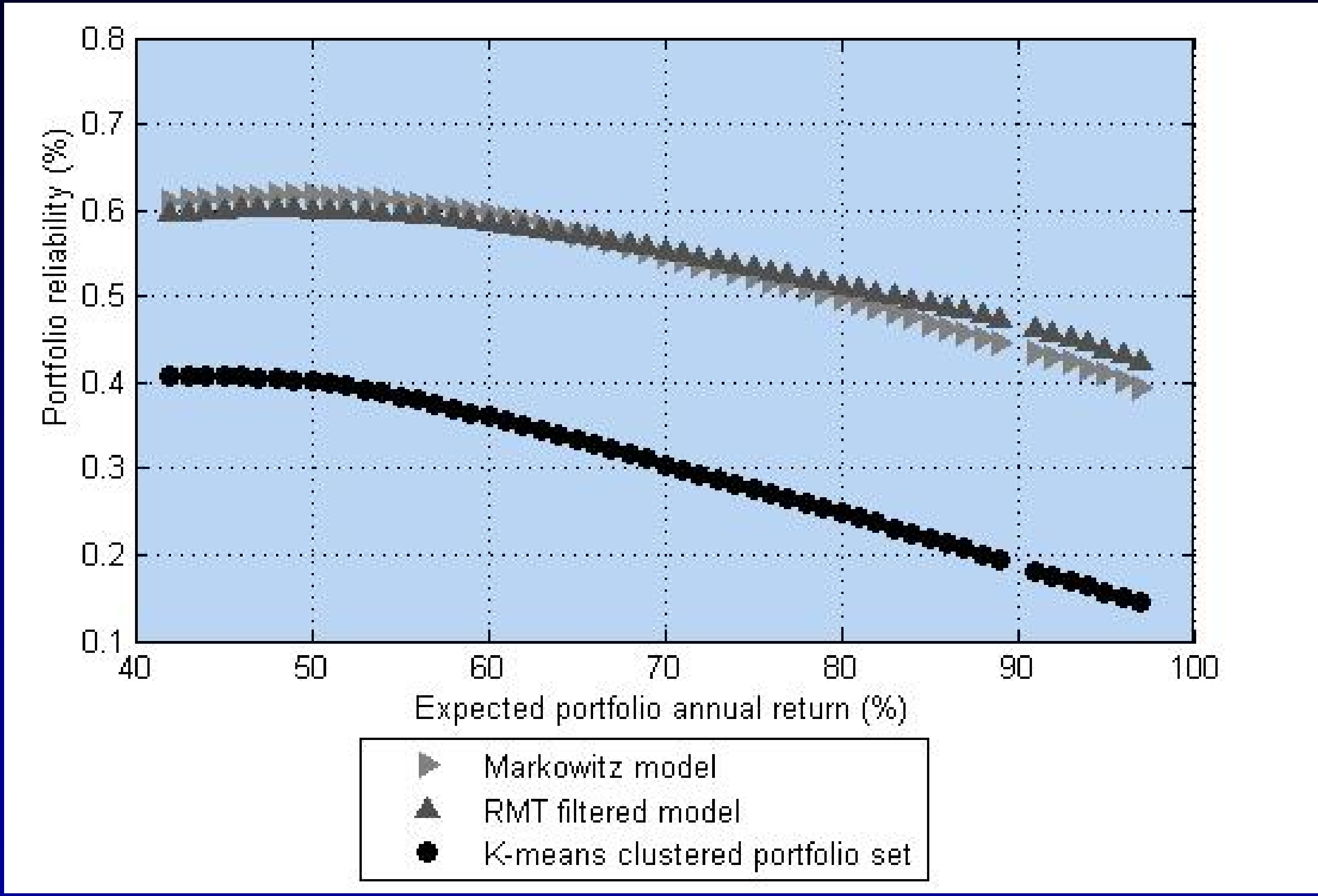


Figure 3(a). A Comparison of the reliabilities of the k-means clustered portfolio set (K=20) with the Markowitz and RMT filtered models of the BSE200 data set.

Performance Analysis

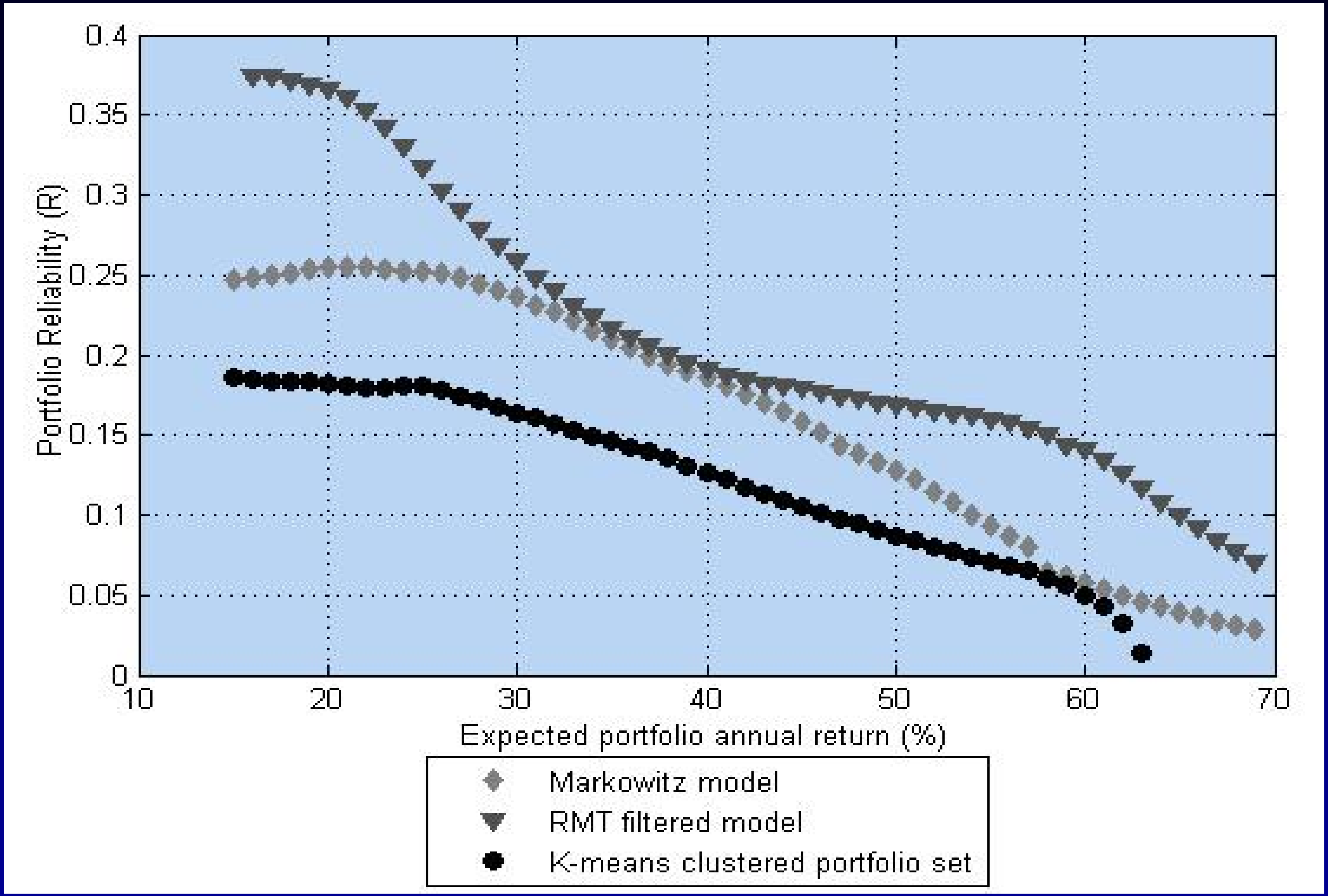


Figure 3(b). A Comparison of the reliabilities of the k-means clustered portfolio set (K=50) with the Markowitz and RMT filtered models of the Nikkei225 data sets.

Performance Analysis

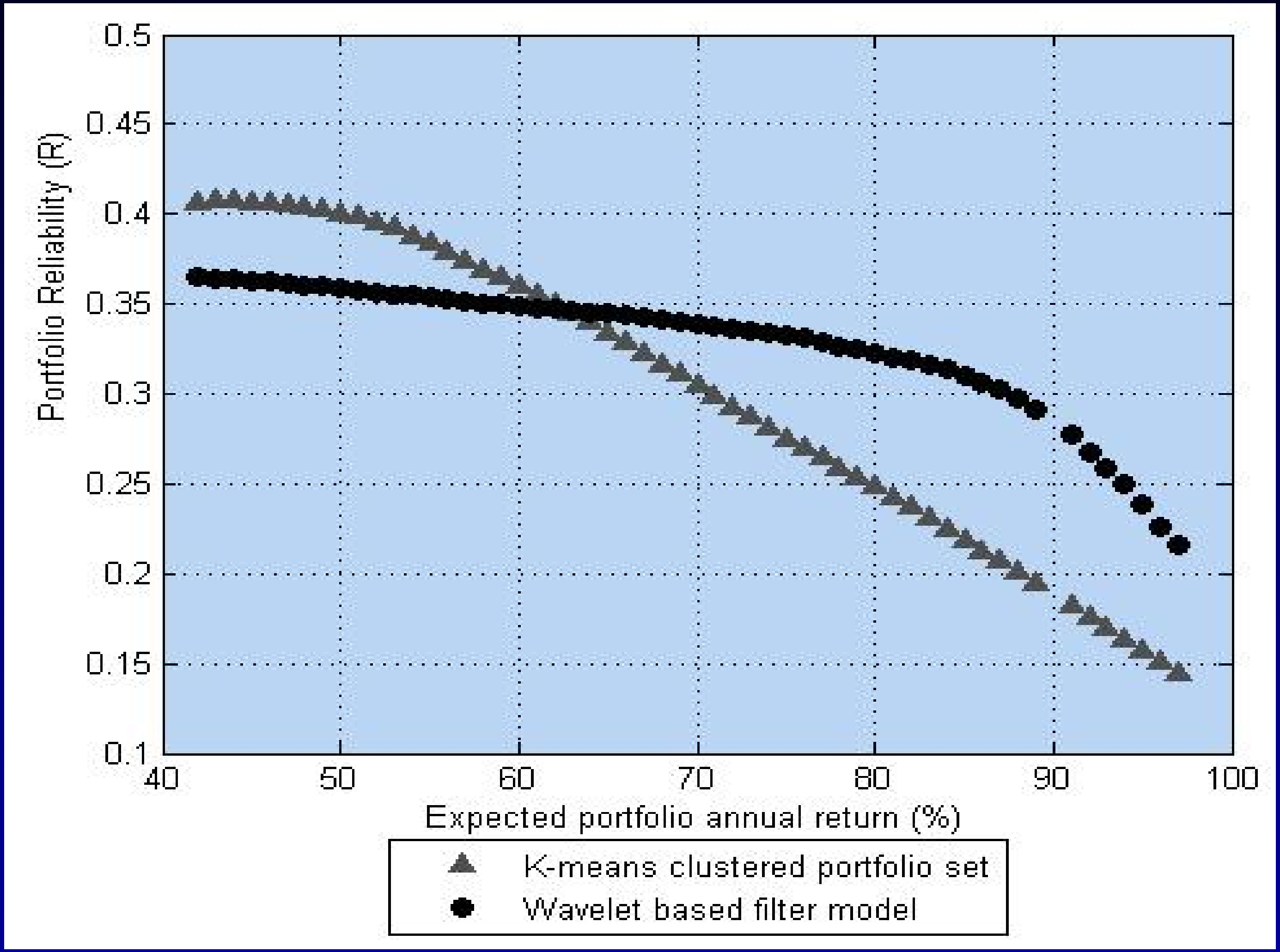


Figure 4(a). A Comparison of the reliabilities of the wavelet based filtering model with the k-means clustered portfolio set (K=20) for the BSE200 data sets.

Performance Analysis

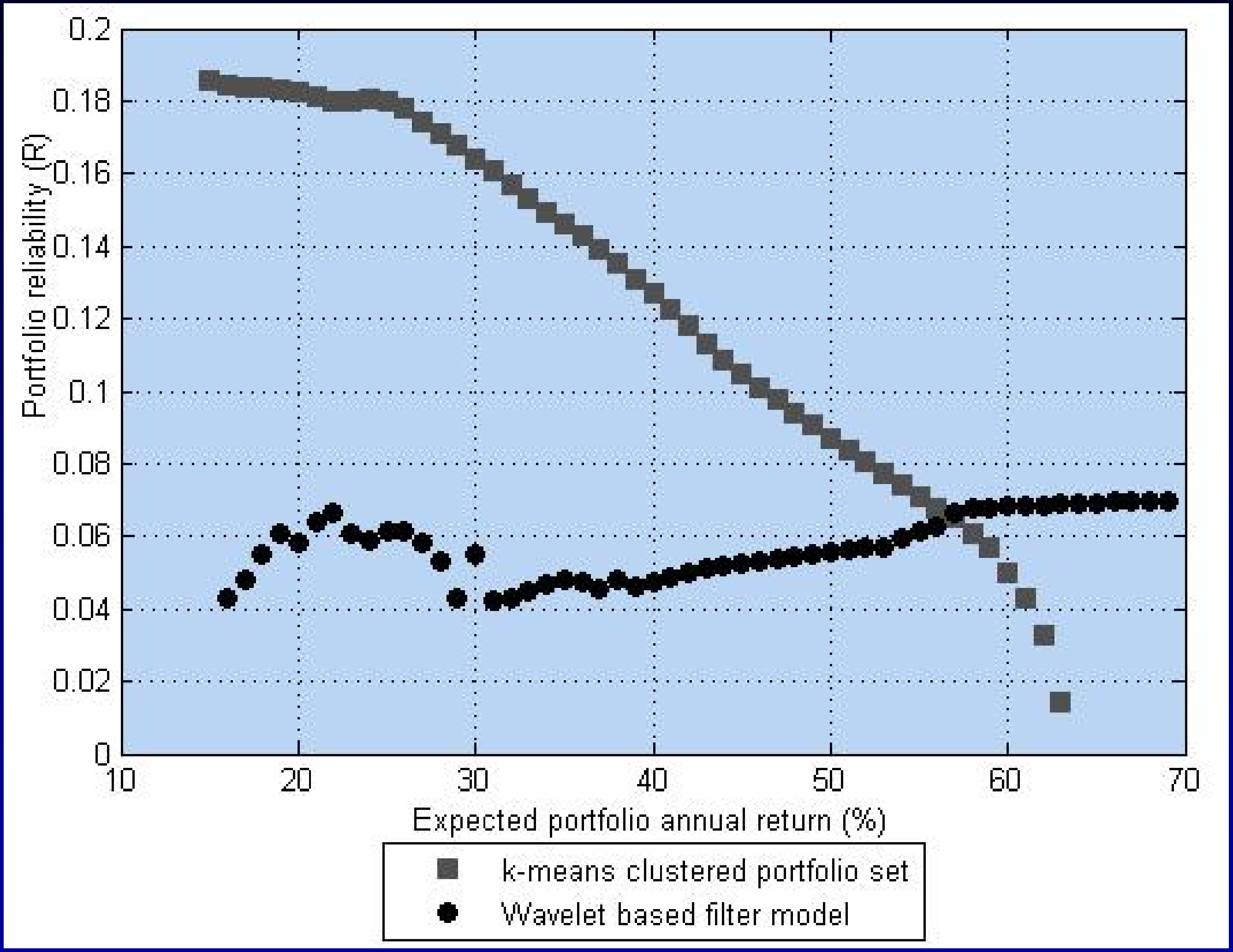


Figure 4(b). A Comparison of the reliabilities of the wavelet based filtering model with the k-means clustered portfolio set (K=50) for the Nikkei225 data sets

Performance Analysis

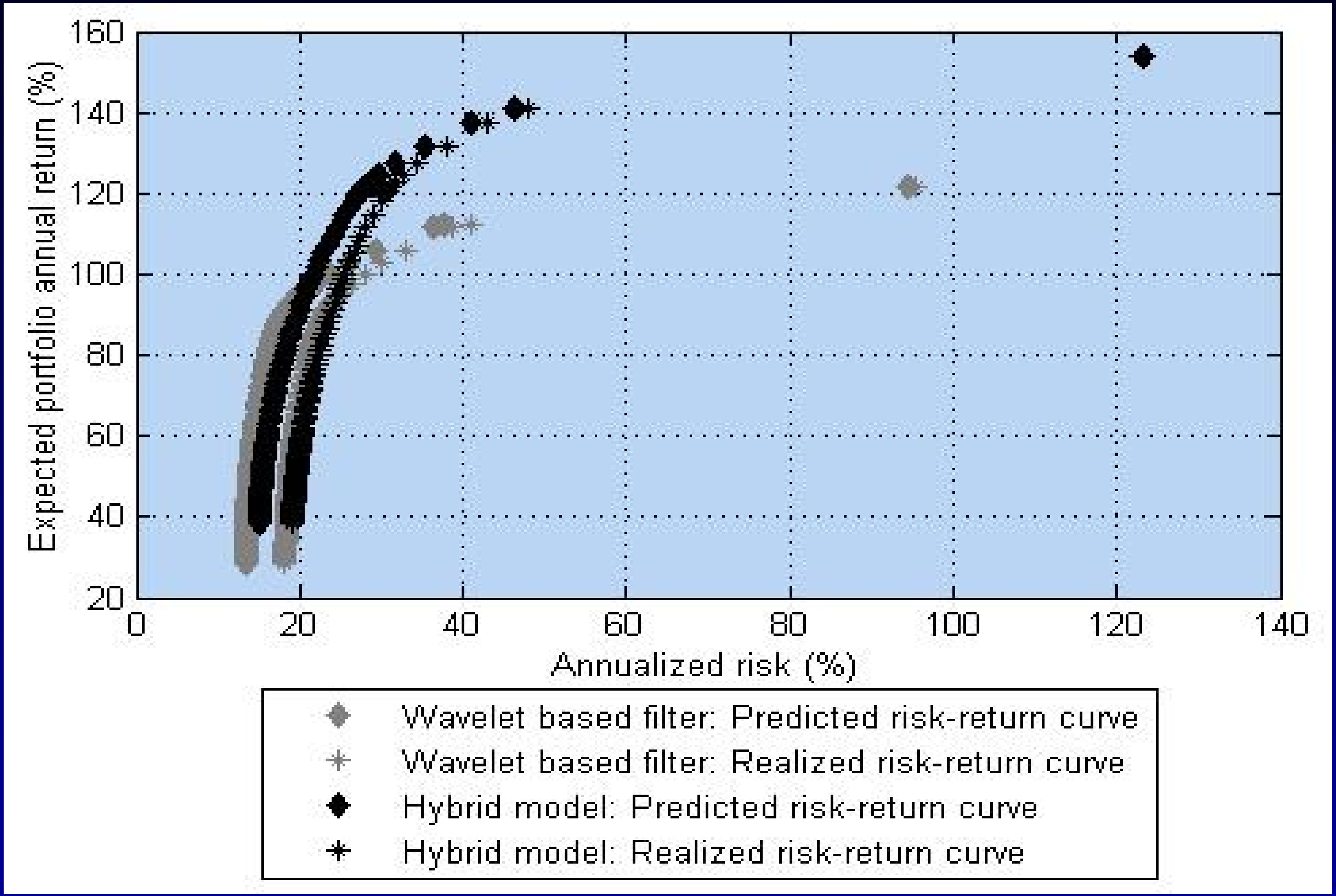


Figure 5(a). Predicted and Realized risk-return graphs for the wavelet based filter and Hybrid model (Wavelet based filter and k-means cluster with K=20) for BSE200 (July 2001 – July 2006) data set.

Performance Analysis

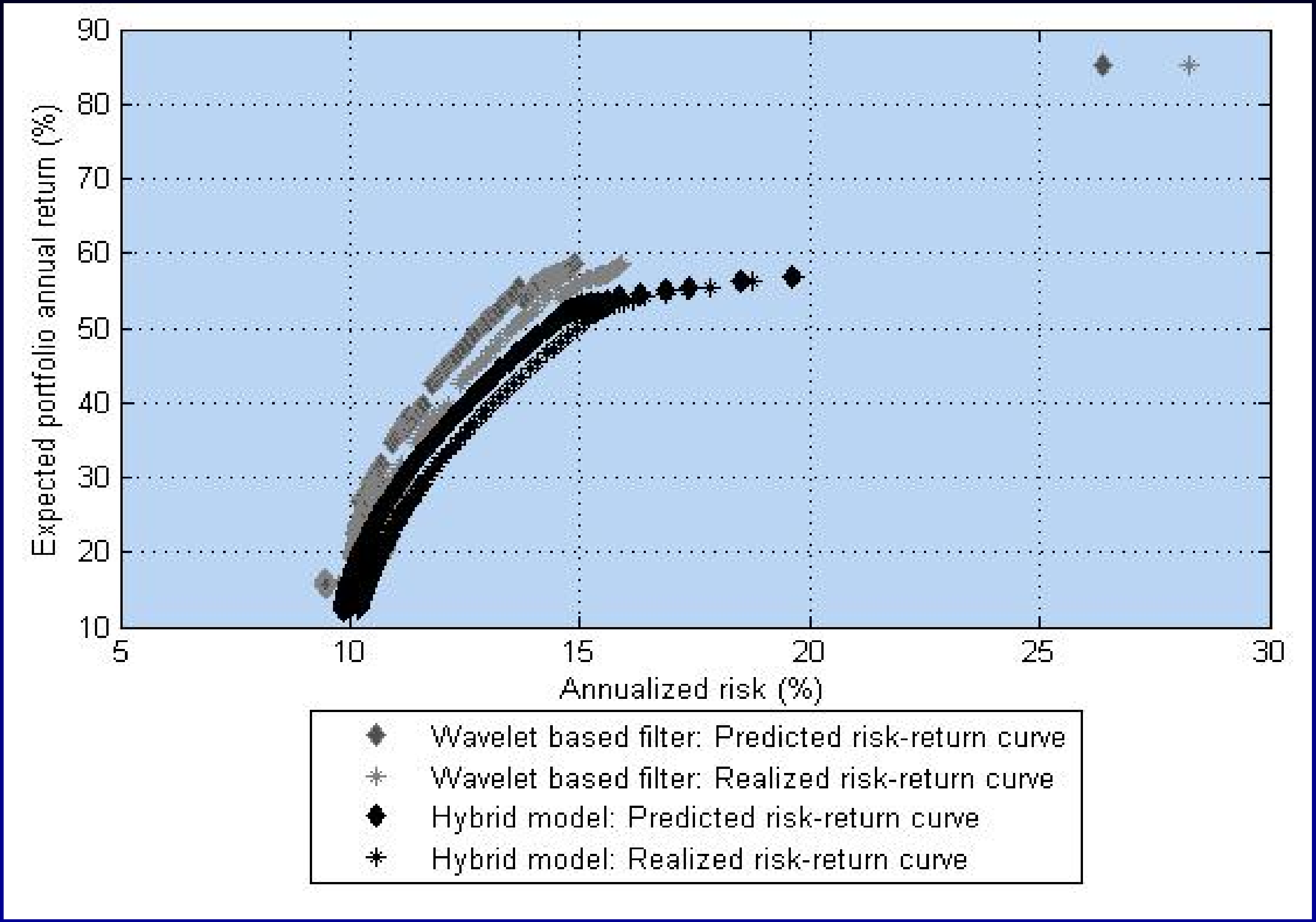


Figure 5(b). Predicted and Realized risk-return graphs for the wavelet based filter and Hybrid model (Wavelet based filter and k-means cluster with K=50) for Nikkei225 (March 2002 – March 2007) data set.

Performance Analysis

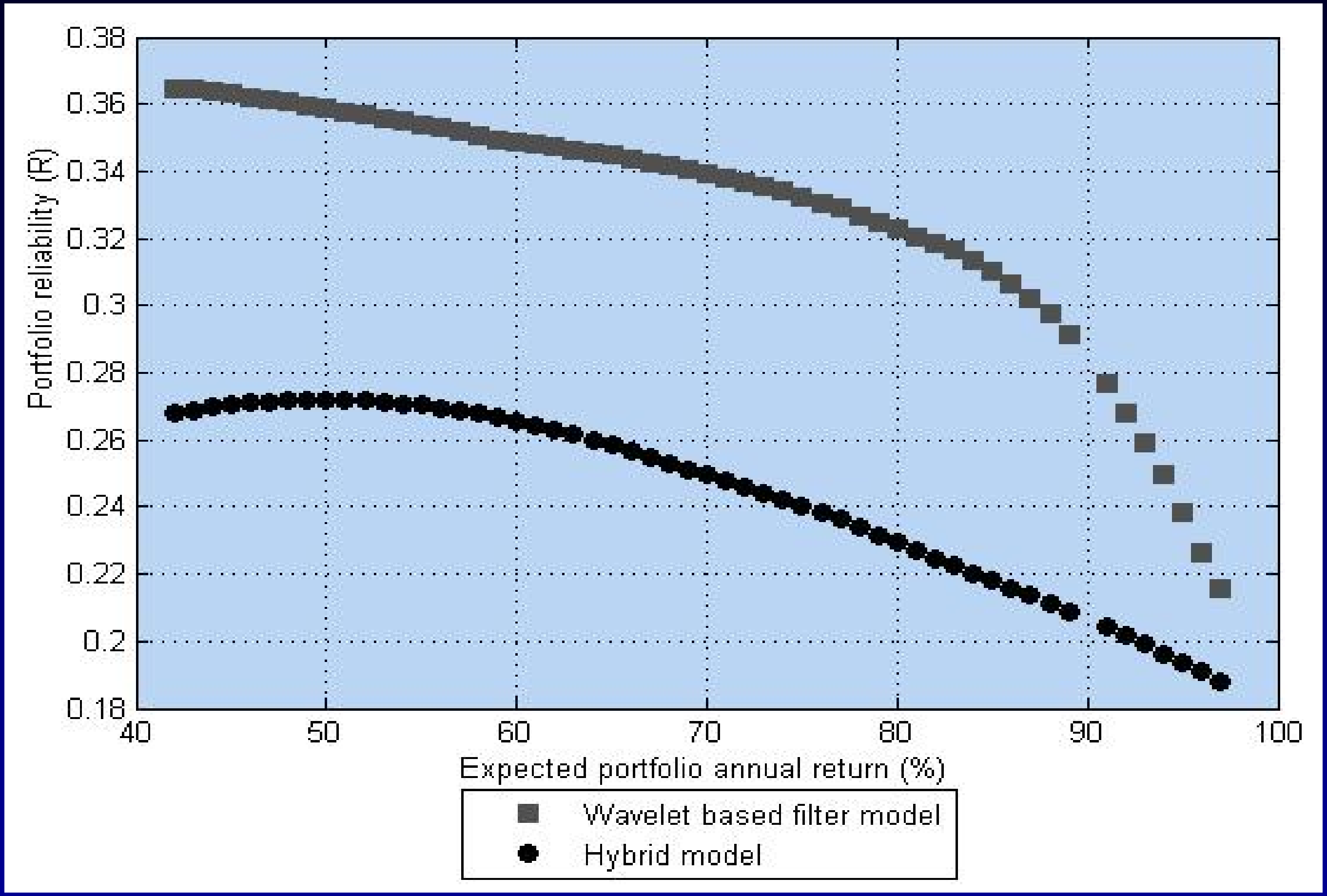


Figure 6(a). A Comparison of the reliabilities of the wavelet based filtering model and hybrid model (K=20) for BSE200 (July 2001 – July 2006) data sets.

Performance Analysis

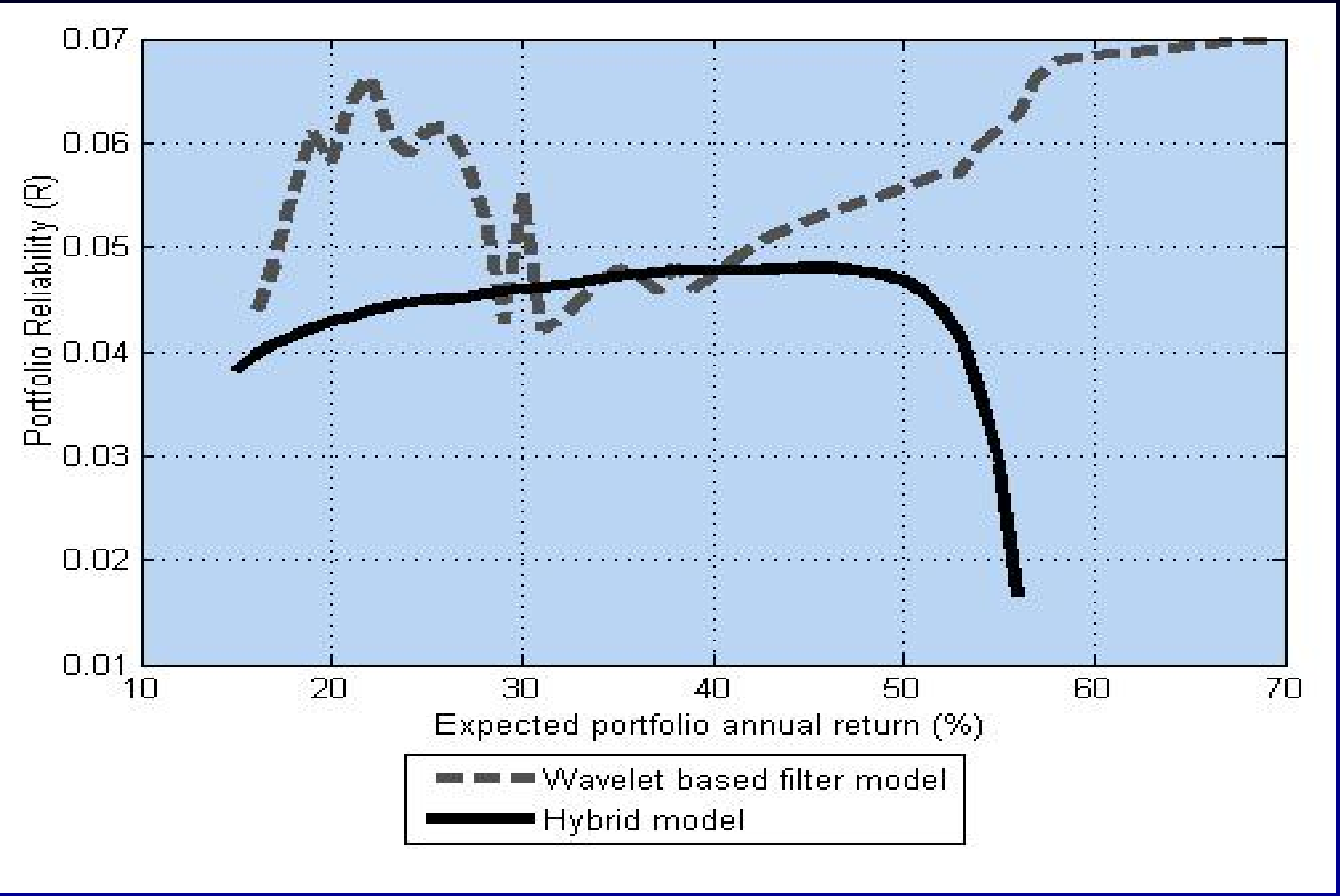


Figure 6(b). A Comparison of the reliabilities of the wavelet based filtering model and hybrid model (K=50) for Nikkei225 (March 2002 – March 2007) data sets

Measuring the efficiency of Expected returns obtained using Hybrid model

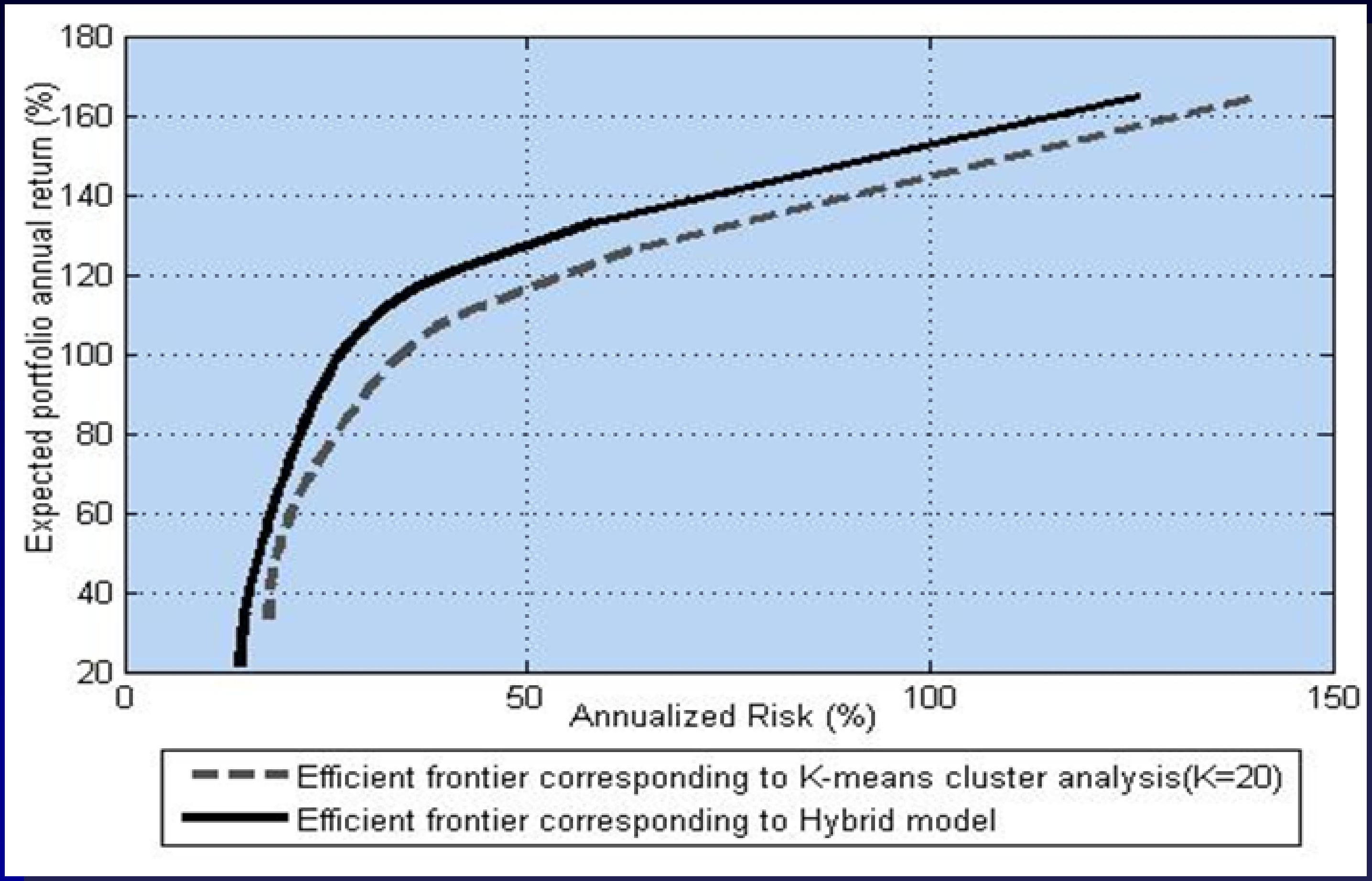


Figure 7. Efficient frontier traced using K-means cluster analysis (K=20) and Hybrid model (Wavelet based filter and k-means cluster filter (K=20)) for BSE200 (July 2001 – July 2006) data sets.

Conclusion

- The hybrid model, which is the sequential combination of wavelet based filter and k-means cluster analysis, serves to yield reliable portfolio sets when compared to Markowitz model, RMT based filter, k-means cluster analysis and also simple wavelet based filter models.

- Inclusion of k-means cluster analysis for the elimination of cardinality constraint promotes dimensionality reduction, i.e., reducing the number of design variables, which could be exploited by any computational approaches, thereby leading to faster convergence.

- Irrespective of diversification in small or large portfolios, the hybrid model based on wavelet filter and k-means cluster analysis yields portfolio sets in realistic time.

- Statistical inference drawn from paired t-test proves that expected returns obtained using hybrid model is better when compared to the expected returns obtained using previous filter models.

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Suggestions

Thank you