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A Study on Ranking of Trapezoidal Intuitionistic Fuzzy Numbers

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Abstract: In this paper attention has been paid to the study of a new ranking procedure for trapezoidal intuitionistic fuzzy number (TRIFN). There are numerous methods for ranking of simple fuzzy numbers but, we lack of effective methods for ranking of intuitionistic fuzzy numbers (IFN). To serve the purpose, the value and ambiguity index of TRIFNs have been defined. In order to rank TRIFNs, we have defined a ranking function by taking sum of value and ambiguity index. To demonstrated our proposed approach one numerical example has been presented.

Keywords: Intuitionistic fuzzy number(IFN), trapezoidal intuitionistic fuzzy number(TRIFN), value index, ambiguity index, ranking of TRIFN.

I. Introduction

Zadeh introduced Fuzzy set theory [50] in 1965. Later on Atanassov generalized the concept of fuzzy set and introduced the idea of Intuitionistic fuzzy set[3],[4],[5]. The concept of fuzzy numbers and arithmetic operations with these numbers were first introduced and investigated by Chang and Zadeh [9] and others. There exist a large amount of literature involving the ranking of fuzzy numbers[1],[2],[10],[11],[20] and decision making with fuzzy numbers[14]-[16]. The notions of intuitionistic fuzzy numbers in different context were studied in [23]-[27],[29],[30],[33],[35],[36],[41],[43] and applied in multi criteria decision making problems [29]-[32],[41]. Intuitionistic fuzzy sets (IFSs), characterized by three functions expressing degrees of membership, non-membership, and hesitation (or indeterminacy), have received increasing attention since their introduction by Atanassov [3[-[5].

Atanassov and Gargov [6] proposed the notion of the interval-valued intuitionistic fuzzy set (IVIFS), which is characterized by a membership function and a non-membership function whose values are intervals rather than exact numbers. Based on IVIFS, Xu [43] defined the notion of interval-valued intuitionistic fuzzy number (IVIFN) and introduced some operations on IVIFNs. IVIFNs have huge amount of application in decision making processes [12],[13],[19],[22],[28]. Further developments of IFS theory, including intuitionistic fuzzy geometry, intuitionistic fuzzy topology, intuitionistic fuzzy logic, an intuitionistic fuzzy approach to artificial intelligence, and intuitionistic fuzzy generalized nets can be found in[37]. Since they are very useful and powerful tool in modeling imprecision or uncertainty, valuable applications of IFSs have been developed in many different fields, including pattern recognition [23],[34], medical diagnosis [17], drug selection [21], microelectronic fault analysis [25],[38], weight assessment [39], and decision-making problems [24],[26]-[29],[41],[44],[49]. The ranking of intuitionistic fuzzy numbers plays a main role in real life problems involving intuitionistic fuzzy decision-making, intuitionistic fuzzy clustering. Recently few methods for ranking IFNs has also been introduced [29],[33],[36],[41],[49]. Wang and Zhang[41] defined the trapezoidal Intuitionistic fuzzy number (TRIFN) and gave ranking method which transformed the ranking of TRIFNs in to ranking of interval numbers. Other than Wang and Zhang, Mitchell[33] introduced ranking of IFNs. Nayagam et al.[36] described Ifns of special type and introduced a method of IF scoring of IFNs as a generalization of Chen and Hwang's scoring for ranking IFNs. In many applications, ranking of interval-valued intuitionistic fuzzy numbers is an important component of the decision-making process. Xu[44] and Xu and Chen [45] proposed score function and accuracy function to rank interval valued intuitionistic fuzzy numbers. Ye [49] also proposed a novel accuracy function to rank interval-valued intuitionistic fuzzy numbers. However in some cases the proposed techniques for ranking interval-valued intuitionistic fuzzy numbers using a score function or an accuracy function do not give sufficient information about alternatives. Li [29] defined the value index and the ambiguity index of triangular intuitionistic fuzzy numbers and developed a ratio ranking method for solving multi-attribute decision-making problems. Though notion of TRIFNs have been established earlier, in this paper we have tried to introduce TRIFNs in a more simplified way which is easy to handle and has a natural interpretation. Furthermore, a new ranking procedure for ranking TRIFNs has been introduced.

The remainder of the paper is organised as follows. In Section 2, we first define TRIFNs, and arithmetic operations of TRIFNs followed by definitions of cut sets. Section 3 defines value and ambiguity indices for membership and non-membership function which is further followed by proposed ranking method. Section 4 contains the conclusion.

II. Basic Definitions

◆ The definition and operations of TRIFNs

Definition1 A TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a special Intuitionistic Fuzzy set on a set of real number **R**, whose membership function and non-membership function are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} w_{\tilde{a}} & a_1 \le x \le a_2 \\ w_{\tilde{a}} & a_2 \le x \le a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)} w_{\tilde{a}} & a_4 \le x \le a_4 \\ 0 & a_4 < x \,\&\, a_1 > x \end{cases}$$

&

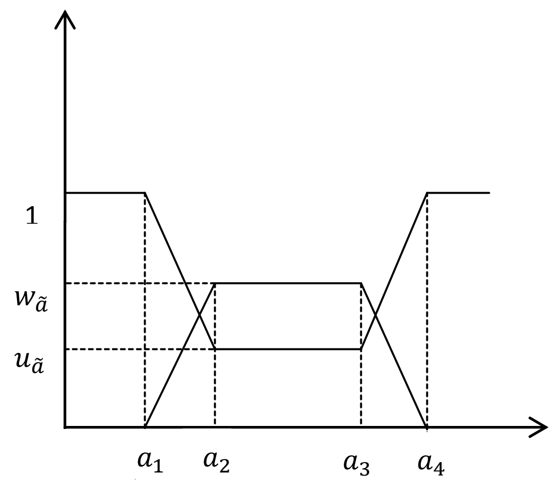
$$\nu_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2-x)+u_{\tilde{a}}(x-a_1)}{(a_2-a_1)} & a_1 \le x \le a_2 \\ u_{\tilde{a}} & a_2 \le x \le a_3 \\ \dfrac{(x-a_3)+u_{\tilde{a}}(a_4-x)}{(a_4-a_3)} & a_4 \le x \le a_4 \\ 1 & a_4 < x \,\&\, a_1 > x \end{cases}$$

respectively as given in Figure 1. The values $w_{\tilde{a}}$ and $u_{\tilde{a}}$ represents the maximum degree of membership and minimum degree of non -membership, respectively, such that the conditions $0 \le w_{\tilde{a}} \le 1$, $0 \le u_{\tilde{a}} \le 1$ $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$ are satisfied. The pa-rameters $w_{\tilde{a}}$ and $u_{\tilde{a}}$ reflects the confidence level and non-confidence level of the TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$, respectively.

Let $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - \nu_{\tilde{a}}(x)$, which is called an IF index of an element x in $\tilde{a}$. It is the degree of the indeterminacy membership of the element x in $\tilde{a}$.

If $a_1 \ge 0$ and one of the four values a_1, a_2, a_3, a_4 is not equal to 0, then the TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is called a positive TRIFN, denoted by $\tilde{a} > 0$.

Likewise, if $a_1 \le 0$ and one of the four values a_1, a_2, a_3, a_4 is not equal to zero, then the TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is called a negative TRIFN, denoted by $\tilde{a} < 0$.



It can be easily seen that $\mu_{\tilde{a}}(x) + \nu_{\tilde{a}}(x) = 1$ for any $x \in R$ if $w_{\tilde{a}} = 1$ and $u_{\tilde{a}} = 0$ which reduces $\tilde{a}$ to $\tilde{a} = \langle (a_1, a_2, a_3, a_4); 1, 0 \rangle$, a trapezoidal fuzzy number. Since the concept of TRIFN is a generalization of that of the trapezoidal fuzzy number, the arithmetical operations of TRIFNs [4] can be defined in the way similar to that of trapezoidal fuzzy numbers as follows:

Definition 2 Let $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3, b_4); w_{\tilde{b}}, u_{\tilde{b}} \rangle$ be two TRIFNs and λ be a real number. The arithmetical operations are listed as follows:

$$\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle$$

$$\tilde{a} - \tilde{b} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\} \rangle$$

$$\tilde{a}\tilde{b}=\begin{cases}\langle(a_1b_1,a_2b_2,a_3b_3,a_4b_4);\min\{w_{\tilde{a}},w_{\tilde{b}}\},\max\{u_{\tilde{a}},u_{\tilde{b}}\}\rangle; \tilde{a}>0,\tilde{b}>0\\ \langle(a_1b_4,a_2b_3,a_3b_2,a_4b_1);\min\{w_{\tilde{a}},w_{\tilde{b}}\},\max\{u_{\tilde{a}},u_{\tilde{b}}\}\rangle; \tilde{a}<0,\tilde{b}>0\\ \langle(a_4b_4,a_3b_3,a_2b_2,a_1b_1);\min\{w_{\tilde{a}},w_{\tilde{b}}\},\max\{u_{\tilde{a}},u_{\tilde{b}}\}\rangle; \tilde{a}<0,\tilde{b}<0\end{cases}$$

$$\lambda\tilde{a}=\begin{cases}\langle(\lambda a_1,\lambda a_2,\lambda a_3,\lambda a_4);w_{\tilde{a}},u_{\tilde{a}}\rangle & ;\lambda>0\\ \langle(\lambda a_1,\lambda a_2,\lambda a_3,\lambda a_4);w_{\tilde{a}},u_{\tilde{a}}\rangle & ;\lambda<0\end{cases}$$

$$\tilde{a}^{-1}=\langle(1/a_1,1/a_2,1/a_3,1/a_4);w_{\tilde{a}},u_{\tilde{a}}\rangle$$

◆ *α -cut sets and β -cut sets of Trapezoidal Intuitionistic fuzzy number (TRIFN):*

Definition 3 [4] A α - cut set of a TRIFN $\tilde{a}=\langle(a_1,a_2,a_3,a_4);w_{\tilde{a}},u_{\tilde{a}}\rangle$ is a crisp subset of R defined as

$$\tilde{a}_\alpha=\{x|\mu_{\tilde{a}}(x)\geq\alpha\} \text{ where } 0\leq\alpha\leq w_{\tilde{a}}.$$

Definition 4 [4] A β - cut set of a TRIFN $\tilde{a}=\langle(a_1,a_2,a_3,a_4);w_{\tilde{a}},u_{\tilde{a}}\rangle$ is a crisp subset of R defined as

$$\tilde{a}_\beta=\{x|\nu_{\tilde{a}}(x)\leq\beta\} \text{ where } u_{\tilde{a}}\leq\beta\leq1.$$

It can be easily followed from the definition of TRIFN , Definition 3 and Definition 4 that $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ are both closed sets and are denoted by $\tilde{a}_\alpha=[L_{\tilde{a}}(\alpha),R_{\tilde{a}}(\alpha)]$ & $\tilde{a}_\beta=[L_{\tilde{a}}(\beta),R_{\tilde{a}}(\beta)]$ respectively. The respective values of $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ are calculated as follows:

$$[L_{\tilde{a}}(\alpha),R_{\tilde{a}}(\alpha)]=\left[a_1+\frac{\alpha(a_2-a_1)}{w_{\tilde{a}}},a_4-\frac{\alpha(a_4-a_3)}{w_{\tilde{a}}}\right]$$

$$[L_{\tilde{a}}(\beta),R_{\tilde{a}}(\beta)]=\left[\frac{(1-\beta)a_2+(\beta-u_{\tilde{a}})a_1}{w_{\tilde{a}}},\frac{(1-\beta)a_3+(\beta-u_{\tilde{a}})a_4}{w_{\tilde{a}}}\right]$$

◆ *Support of a TRIFN*

Definition 5 The Support of a TRIFN $\tilde{a}=\langle(a_1,a_2,a_3,a_4);w_{\tilde{a}},u_{\tilde{a}}\rangle$ for the membership function is defined as

$$\operatorname{supp}_\mu(\tilde{a})=\{x|\mu_{\tilde{a}}(x)\geq0\}.$$

Definition 6 The Support of a TRIFN $\tilde{a}=\langle(a_1,a_2,a_3,a_4);w_{\tilde{a}},u_{\tilde{a}}\rangle$ for the non-membership function is defined as

$$\operatorname{supp}_\nu(\tilde{a})=\{x|\nu_{\tilde{a}}(x)\leq1\}.$$

III. Ranking of TRIFNs based on Value and Ambiguity

◆ *Value and ambiguity of a TRIFN*

The value and ambiguity of a TRIFN can be defined similarly to those of a TIFNs introduced by D.F.Li [29].

Definition 7 Let $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ be an α -cut set and a β -cut set of a TRIFN $\tilde{a}=\langle(a_1,a_2,a_3,a_4);w_{\tilde{a}},u_{\tilde{a}}\rangle$, respectively. Then the values of the membership function $\mu_{\tilde{a}}(x)$ and the non-membership function $\nu_{\tilde{a}}(x)$ for the TRIFN $\tilde{a}$ are defined as follows:

$$V_\mu(\tilde{a})=\int_0^{w_{\tilde{a}}}\frac{L_{\tilde{a}}(\alpha)+R_{\tilde{a}}(\alpha)}{2}f(\alpha)d\alpha$$

$$V_\nu(\tilde{a})=\int_{u_{\tilde{a}}}^{1}\frac{L_{\tilde{a}}(\beta)+R_{\tilde{a}}(\beta)}{2}g(\beta)d\beta$$

respectively, where the function $f(\alpha)$ is a non-negative and non-decreasing function on the interval $[0,w_{\tilde{a}}]$ with $f(0)=0$ and $\int_0^{w_{\tilde{a}}}f(\alpha)d\alpha=w_{\tilde{a}}$; the function $g(\beta)$ is a non-negative and non-increasing function on the interval $[u_{\tilde{a}},1]$ with $g(1)=0$ and $\int_{u_{\tilde{a}}}^{1}g(\beta)d\beta=1-u_{\tilde{a}}$.

Throughout the paper we shall choose $f(\alpha)=\frac{2\alpha}{w_{\tilde{a}}}$, $\alpha\in[0,w_{\tilde{a}}]$ $g(\beta)=\frac{2(1-\beta)}{1-u_{\tilde{a}}}$, $\beta\in[u_{\tilde{a}},1]$

The value of membership and non-membership of a TRIFN $\tilde{a}_\alpha$ can be derived in analogy with value of membership and non-membership for TIFN [29].

The value of the membership function of a TRIFN $\tilde{a}$ is calculated as follows:

$$V_\mu(\tilde{a})=\int_0^{w_{\tilde{a}}}\left[a_1+\frac{\alpha(a_2-a_1)}{w_{\tilde{a}}}+a_4-\frac{\alpha(a_4-a_3)}{w_{\tilde{a}}}\right]\frac{\alpha}{w_{\tilde{a}}}d\alpha$$

$$=\left[\frac{a_1+a_4}{2w_{\tilde{a}}}\alpha^2\right]_0^{w_{\tilde{a}}}+\left[\frac{a_2-a_1-a_4+a_3}{3(w_{\tilde{a}})^2}\alpha^3\right]_0^{w_{\tilde{a}}}$$

$$=\frac{a_1+a_4+2(a_2+a_3)}{6}w_{\tilde{a}}$$

In a similar way value of non-membership can be evaluated as follows:

$$V_\nu(\tilde{a}) = \int_{u_{\tilde{a}}}^{1}\left[\frac{(1-\beta)a_2+(\beta-u_{\tilde{a}})a_1}{w_{\tilde{a}}}+\frac{(1-\beta)a_3+(\beta-u_{\tilde{a}})a_4}{w_{\tilde{a}}}\right]\frac{1-\beta}{1-u_{\tilde{a}}}d\beta$$

$$= \int_{u_{\tilde{a}}}^{1}\frac{(a_2+a_3-a_1-a_4)(1-\beta)^2+(1-u_{\tilde{a}})(a_1+a_4)(1-\beta)}{(1-u_{\tilde{a}})}d\beta$$

$$= -\left[\frac{(a_2+a_3-a_1-a_4)(1-\beta)^3}{3(1-u_{\tilde{a}})^2}\right]_{u_{\tilde{a}}}^{1} - \left[\frac{(1-u_{\tilde{a}})(a_1+a_4)(1-\beta)^2}{2(1-u_{\tilde{a}})^2}\right]_{u_{\tilde{a}}}^{1}$$

$$= \frac{[a_1+a_4+2(a_2+a_3)](1-u_{\tilde{a}})}{6}.$$

With the condition that $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$, it follows that $V_\mu(\tilde{a}) \le V_\nu(\tilde{a})$. Thus, the values of membership and non-membership functions of a TRIFN $\tilde{a}$ may be concisely expressed as an interval $[V_\mu(\tilde{a}), V_\nu(\tilde{a})]$. The following theorem describes the linearity property of value of membership and non-membership as well:

Theorem 1 Let $\tilde{a} = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}\rangle$ and $\tilde{b} = \langle(b_1, b_2, b_3, b_4); w_{\tilde{b}}, u_{\tilde{b}}\rangle$ be two TRIFNs with $w_{\tilde{a}} = u_{\tilde{a}}$ and $w_{\tilde{b}} = u_{\tilde{b}}$ .Then, we have $V_\mu(\tilde{a}+\tilde{b}) = V_\mu(\tilde{a}) + V_\mu(\tilde{b})$ and $V_\nu(\tilde{a}+\tilde{b}) = V_\nu(\tilde{a}) + V_\nu(\tilde{b})$

Proof: We have

$$\tilde{a}+\tilde{b} = \left\langle(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\}\right\rangle$$

Now, $V_\mu(\tilde{a}+\tilde{b}) = \dfrac{[(a_1+b_1)+2(a_2+b_2)+2(a_3+b_3)+(a_4+b_4)]w_{\tilde{a}}}{6}$

$$= \frac{a_1+a_4+2(a_2+a_3)}{6}w_{\tilde{a}} + \frac{b_1+b_4+2(b_2+b_3)}{6}w_{\tilde{b}}$$

$$= V_\mu(\tilde{a}) + V_\mu(\tilde{b})$$

Similarly,

$$V_\nu(\tilde{a}+\tilde{b}) = \frac{[(a_1+b_1)+2(a_2+b_2)+2(a_3+b_3)+(a_4+b_4)](1-u_{\tilde{a}})}{6}$$

$$= \frac{a_1+a_4+2(a_2+a_3)}{6}(1-u_{\tilde{a}}) + \frac{b_1+b_4+2(b_2+b_3)}{6}(1-u_{\tilde{b}})$$

$$= V_\nu(\tilde{a}) + V_\nu(\tilde{b})$$

Hence, the proof.

Definition 8 Let $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ be an α -cut set and a β -cut set of a TRIFN $\tilde{a} = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}\rangle$, respectively. Then the ambiguities of the membership function $\mu_{\tilde{a}}(x)$ and the non-membership function $\nu_{\tilde{a}}(x)$ for the TRIFN $\tilde{a}$ are defined as follows:

$$A_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}}(R_{\tilde{a}}(\alpha) - L_{\tilde{a}}(\alpha))f(\alpha)d\alpha$$

$$A_\nu(\tilde{a}) = \int_{u_{\tilde{a}}}^{1}(R_{\tilde{a}}(\beta) - L_{\tilde{a}}(\beta))g(\beta)d\beta$$

respectively. It can be easily followed from Definition of $A_\mu(\tilde{a})$ and $A_\nu(\tilde{a})$ that $A_\mu(\tilde{a}) \ge 0$, $A_\nu(\tilde{a}) \ge 0$.

The ambiguity of the membership function of a TRIFN $\tilde{a}$ is calculated as follows:

$$A_\mu(\tilde{a}) = \int_0^{w_{\tilde{a}}}\left[a_4 - \frac{\alpha(a_4-a_3)}{w_{\tilde{a}}} - a_1 - \frac{\alpha(a_2-a_1)}{w_{\tilde{a}}}\right]\frac{2\alpha}{w_{\tilde{a}}}d\alpha$$

$$= \left[\frac{a_4-a_1}{w_{\tilde{a}}}\alpha^2\right]_0^{w_{\tilde{a}}} - 2\left[\frac{a_2-a_1+a_4-a_3}{3(w_{\tilde{a}})^2}\alpha^3\right]_0^{w_{\tilde{a}}}$$

$$= \frac{(a_4-a_1)-2(a_2-a_3)}{3}w_{\tilde{a}}$$

Similarly, the ambiguity of the non-membership function of a TRIFN $\tilde{a}$ is given by:

$$A_\nu(\tilde{a}) = \int_{u_{\tilde{a}}}^{1}\left[\frac{(1-\beta)a_3+(\beta-u_{\tilde{a}})a_4}{1-u_{\tilde{a}}} - \frac{(1-\beta)a_2+(\beta-u_{\tilde{a}})a_1}{1-u_{\tilde{a}}}\right]\frac{2(1-\beta)}{1-u_{\tilde{a}}}d\beta$$

$$= \int_{u_{\tilde{a}}}^{1}\left[\frac{2[(a_2-a_3-a_1+a_4)(1-\beta)^2+(1-u_{\tilde{a}})(1-\beta)(a_4-a_1)]}{(1-u_{\tilde{a}})^2}\right]d\beta$$

$$= \left[\frac{2(a_4-a_1+a_2-a_3)(1-\beta)^3}{3(1-u_{\tilde{a}})^2}\right]_{u_{\tilde{a}}}^{1} - \left[\frac{(a_4-a_1)(1-u_{\tilde{a}})(1-\beta)^2}{(1-u_{\tilde{a}})^2}\right]_{u_{\tilde{a}}}^{1}$$

$$= \frac{(a_4-a_1)-2(a_2-a_3)}{3}(1-u_{\tilde{a}})$$

With the condition that $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$,it follows that $A_\mu(\tilde{a}) \le A_\nu(\tilde{a})$. Thus, the ambiguities of membership and non-membership functions of a TRIFN $\tilde{a}$ may be concisely expressed as an interval $[A_\mu(\tilde{a}), A_\nu(\tilde{a})]$.

From the definition of ambiguities of the membership and non-membership function of a TRIFN ,it readily follows that :

Theorem 2 Let $\tilde{a} = \langle(a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}\rangle$ and $\tilde{b} = \langle(b_1, b_2, b_3, b_4); w_{\tilde{b}}, u_{\tilde{b}}\rangle$ be two TRIFNs with $w_{\tilde{a}} = u_{\tilde{a}}$ and $w_{\tilde{b}} = u_{\tilde{b}}$. Then, we have $A_\mu(\tilde{a}+\tilde{b}) = A_\mu(\tilde{a}) + A_\mu(\tilde{b})$ and $A_\nu(\tilde{a}+\tilde{b}) = A_\nu(\tilde{a}) + A_\nu(\tilde{b})$

Proof: We have

$\tilde{a}+\tilde{b} = \left\langle(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4); \min\{w_{\tilde{a}}, w_{\tilde{b}}\}, \max\{u_{\tilde{a}}, u_{\tilde{b}}\}\right\rangle$ From the definition of ambiguity of membership function,

$$A_\mu(\tilde{a}+\tilde{b}) = \frac{(a_4+b_4)-(a_1+b_1)-2((a_2+b_2)-(a_3+b_3))}{3}w_{\tilde{a}}$$

$$= \frac{(a_4-a_1)-2(a_2-a_3)}{3}w_{\tilde{a}} + \frac{(b_4-b_1)-2(b_2-b_3)}{3}w_{\tilde{a}}$$

$$= A_\mu(\tilde{a}) + A_\mu(\tilde{b})$$

Similarly,

$$A_\nu(\tilde a+\tilde b)=\frac{(a_4+b_4)-(a_1+b_1)-2((a_2+b_2)-(a_3+b_3))}{3}(1-u_{\tilde a})$$
$$=\frac{(a_4-a_1)-2(a_2-a_3)}{3}(1-u_{\tilde a})+\frac{(b_4-b_1)-2(b_2-b_3)}{3}(1-u_{\tilde a})$$
$$=A_\nu(\tilde a)+A_\nu(\tilde b)$$

Hence, the proof.

◆ *The proposed ranking method*

In this Section, a new ranking method is introduced by taking the sum of value index and ambiguity index. Earlier, a ranking method for TIFNs has been introduced by D.F.Li [29] which was based on ratio of value and ambiguity index. Though the ranking procedure introduced by Li was more generalized and applicability was more wide, but the ratio ranking method lack in linearity property. It can be easily seen that for ratio ranking method $R(\tilde a+\tilde b,\lambda)\neq R(\tilde a,\lambda)+R(\tilde b,\lambda)$. In our proposed method, we have tried to rectify this by taking linear sum of value and ambiguity indices.

Definition 9 Let $\tilde a=\langle(a_1,a_2,a_3,a_4);w_{\tilde a},u_{\tilde a}\rangle$ be a TRIFN. A value index and ambiguity index for the TRIFN $\tilde a$ are defined as follows:

$$V(\tilde a,\lambda)=V_\mu(\tilde a)+\lambda(V_\nu(\tilde a)-V_\mu(\tilde a))$$
$$A(\tilde a,\lambda)=A_\nu(\tilde a)+\lambda(A_\nu(\tilde a)-A_\mu(\tilde a))$$

respectively, where $\lambda\in[0,1]$ is a weight which represents the decision maker's preference information. Limited to the above formulation, the choice $\lambda=\frac{1}{2}$ appears to be a reasonable one. One can choose λ according to the suitability of the subject. $\lambda\in[0,\frac{1}{2})$ indicates decision maker's pessimistic attitude towards uncertainty while $\lambda\in(\frac{1}{2},1]$ indicates decision maker's optimistic attitude towards uncertainity.

With our choice $\lambda=\frac{1}{2}$, the value and ambiguity indices for TRIFN reduces to the following:

$$V(\tilde a,\frac{1}{2})=\frac{V_\mu(\tilde a)+V_\nu(\tilde a)}{2}\quad A(\tilde a,\frac{1}{2})=\frac{A_\mu(\tilde a)+A_\nu(\tilde a)}{2}$$

The proposed ranking method is as follows:

$$R(\tilde a)=V(\tilde a,\tfrac{1}{2})+A(\tilde a,\tfrac{1}{2})$$

It is quite easy to see that the above proposed ranking method satisfies the seven axioms namely the Reasonable properties proposed by Wang &Kerre [41]. Furthermore, the raking procedure is a two-index ranking approach as value of $R(\tilde a)$ depend upon both $V(\tilde a)$ and $A(\tilde a)$.

Remark: Since we are taking $\lambda=\frac{1}{2}$ throughout the paper, instead of $V(\tilde a,\frac{1}{2})$ and $A(\tilde a,\frac{1}{2})$ we shall write $V(\tilde a)$ and $A(\tilde a)$ respectively.It can be easily seen that the value index $V(\tilde a)$, ambiguity index $A(\tilde a)$ and ranking function $R(\tilde a)$ has the following properties:

Theorem 3 Let $\tilde a=\langle(a_1,a_2,a_3,a_4);w_{\tilde a},u_{\tilde a}\rangle$ and $\tilde b=\langle(b_1,b_2,b_3,b_4);w_{\tilde b},u_{\tilde b}\rangle$ be two TRIFNs with $w_{\tilde a}=u_{\tilde a}$ and $w_{\tilde b}=u_{\tilde b}$ .Then, we have $V(\tilde a+\tilde b)=V(\tilde a)+V(\tilde b)$.

Proof: We have $V(\tilde a+\tilde b)=\frac{V_\mu(\tilde a+\tilde b)+V_\nu(\tilde a+\tilde b)}{2}$

$$=\frac{V_\mu(\tilde a)+V_\mu(\tilde b)+V_\nu(\tilde a)+V_\nu(\tilde b)}{2}$$

[UsingTheorem 1]

$$=\frac{V_\mu(\tilde a)+V_\nu(\tilde a)}{2}+\frac{V_\mu(\tilde b)+V_\nu(\tilde b)}{2}=V(\tilde a)+V(\tilde b)$$

Theorem 4 Let $\tilde a=\langle(a_1,a_2,a_3,a_4);w_{\tilde a},u_{\tilde a}\rangle$ and $\tilde b=\langle(b_1,b_2,b_3,b_4);w_{\tilde b},u_{\tilde b}\rangle$ be two TRIFNs with $w_{\tilde a}=u_{\tilde a}$ and $w_{\tilde b}=u_{\tilde b}$ .Then, we have $A(\tilde a+\tilde b)=A(\tilde a)+A(\tilde b)$.

Proof: We have

$$A(\tilde a+\tilde b)=\frac{A_\mu(\tilde a+\tilde b)+A_\nu(\tilde a+\tilde b)}{2}$$
$$=\frac{A_\mu(\tilde a)+A_\mu(\tilde b)+A_\nu(\tilde a)+A_\nu(\tilde b)}{2}$$

[Using Theorem 2]

$$=\frac{A_\mu(\tilde a)+A_\nu(\tilde a)}{2}+\frac{A_\mu(\tilde b)+A_\nu(\tilde b)}{2}$$
$$=A(\tilde a)+A(\tilde b)$$

Theorem 5 Let $\tilde a=\langle(a_1,a_2,a_3,a_4);w_{\tilde a},u_{\tilde a}\rangle$ and $\tilde b=\langle(b_1,b_2,b_3,b_4);w_{\tilde b},u_{\tilde b}\rangle$ be two TRIFNs with $w_{\tilde a}=u_{\tilde a}$ and $w_{\tilde b}=u_{\tilde b}$ .Then, we have $R(\tilde a+\tilde b)=R(\tilde a)+R(\tilde b)$.

Proof We have $R(\tilde a+\tilde b)=V(\tilde a+\tilde b)+A(\tilde a+\tilde b)$

$$=V(\tilde a)+V(\tilde b)+A(\tilde a)+A(\tilde b)$$

[Using Theorem 3 and 4]

$$=[V(\tilde a)+A(\tilde a)]+[V(\tilde b)+A(\tilde b)]$$
$$=R(\tilde a)+R(\tilde b)$$

Let $\tilde a_j=\left\langle(a_{1j},a_{2j},a_{3j},a_{4j});w_{\tilde a_j},u_{\tilde a_j}\right\rangle$;
(i=1,2,3,4;j=1,2,3,..........N) be N numbers of TRIFNs.

The whole procedure of ranking TRIFN by the proposed method is stated as follows:

(i) Evaluate $V_\mu(\tilde{a}_j), V_\nu(\tilde{a}_j), A_\mu(\tilde{a}_j), A_\nu(\tilde{a}_j)$ as defined in Definition 7 and 8 respectively.
(ii) Evaluate $V(\tilde{a}_j)$ and $A(\tilde{a}_j)$ from Definition 9

(iii) Finally evaluate $R(\tilde{a}_j)$, by taking sum of $V(\tilde{a}_j)$ and $A(\tilde{a}_j)$. Then the ranking of TRIFNs is according to

(a)if $R(\tilde{a}_j) > R(\tilde{a}_k)$,then $\tilde{a}_j > \tilde{a}_k$.

(b)if $R(\tilde{a}_j) < R(\tilde{a}_k)$,then $\tilde{a}_j < \tilde{a}_k$.

(c)if $R(\tilde{a}_j) = R(\tilde{a}_k)$,then $\tilde{a}_j = \tilde{a}_k$.

◆ Numerical example

To illustrate the process of ranking TRIFNs by proposed ranking method, we take three TRIFNs as follows:

Let $\tilde{a}_1 = \langle (0.1,0.3,0.5,0.8);0.5,0.2 \rangle$,
$\tilde{a}_2 = \langle (0.2,0.3,0.6,0.9);0.6,0.4 \rangle$ and
$\tilde{a}_3 = \langle (0.1,0.5,0.7,0.9);0.5,0.3 \rangle$ be three TRIFNs.

Step-(i): $V_\mu(\tilde{a}_j), V_\nu(\tilde{a}_j), A_\mu(\tilde{a}_j), A_\nu(\tilde{a}_j)$ respectively, are evaluated as follows:

$$V_\mu(\tilde{a}_1) = \frac{0.1 + 0.8 + 2(0.3 + 0.5)}{2} \times 0.5$$
$$= 0.4166 \times 0.5 = .2083$$
$$V_\nu(\tilde{a}_1) = .4166 \times 0.8 = 0.3333$$
$$V_\mu(\tilde{a}_2) = .4833 \times 0.6 = 0.29$$
$$V_\nu(\tilde{a}_2) = .4833 \times 0.6 = 0.29$$
$$V_\mu(\tilde{a}_3) = .5666 \times 0.5 = 0.2833$$
$$V_\nu(\tilde{a}_3) = .5666 \times 0.7 = 0.3966$$

Similarly,

$$A_\mu(\tilde{a}_1) = \frac{(0.8 - 0.1) - 2(0.3 - 0.5)}{2} \times 0.5 = 0.3666 \times 0.5 = 0.1833$$
$$A_\nu(\tilde{a}_1) = 0.3666 \times 0.8 = 0.2932$$
$$A_\mu(\tilde{a}_2) = 0.4333 \times 0.6 = 0.26$$
$$A_\nu(\tilde{a}_2) = 0.4333 \times 0.6 = 0.26$$
$$A_\mu(\tilde{a}_3) = 0.4 \times 0.5 = 0.2$$
$$A_\nu(\tilde{a}_3) = 0.4 \times 0.7 = 0.28$$

Step-(ii): The value indices $V(\tilde{a}_j)$ and ambiguity indices $A(\tilde{a}_j)$ for three TRIFNs are calculated as follows:

$$V(\tilde{a}_1) = \frac{0.2803 + 0.3333}{2} = 0.2708$$
$$V(\tilde{a}_2) = 0.29$$
$$V(\tilde{a}_3) = 0.3399$$
$$A(\tilde{a}_1) = \frac{0.1833 + 0.2932}{2} = 0.2382$$
$$A(\tilde{a}_2) = 0.26$$
$$A(\tilde{a}_3) = 0.24$$

Step-(iii): $R(\tilde{a}_j)$ for j=1,2,3 are calculated as follows:

$$R(\tilde{a}_1) = 0.2708 + 0.2382 = 0.509$$
$$R(\tilde{a}_2) = 0.29 + 0.26 = 0.55$$
$$R(\tilde{a}_3) = 0.3399 + 0.24 = 0.5799$$

From the above calculation we can conclude that $R(\tilde{a}_3) > R(\tilde{a}_2) > R(\tilde{a}_1)$ which implies $\tilde{a}_3 > \tilde{a}_2 > \tilde{a}_1$.

IV. Conclusion

In this paper we have defined value and ambiguity indices of TRIFNs in analogy with the definition of value and ambiguity of TIFNs defined by D.F.Li [29]. But instead of ratio of value and ambiguity indices we have taken sum of value and ambiguity indices. It is easy to see that the proposed ranking function satisfies linear property. However, the above proposed ranking method is to implement and evaluate and has a natural interpretation. Moreover, the result obtained by this method also agrees with human intention.

V. References

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