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Mathematical Model based on Game Theory and Markov Chains for Analysing the Transmission Cost in SA-ZD Mechanism

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Abstract: **In Slotted ALOHA (SA), mobile nodes tend to be more aggressive when they compete to get access to the common channel, which leads to a dramatic decreases of the Quality of Service (QoS). In this paper, we analyze the Slotted ALOHA and Slotted ALOHA combined with ZigZag Decoding (SA-ZD) within a stochastic game framework. First, we start with the cooperative model of the proposed mechanism in which we propose several approach to optimize the system. Then, we model the system by a non-cooperative game using a bi-dimensional Markov chain that integrates the effect of ZigZag Decoding (ZD). We then propose an adjustable transmission cost to compromise between throughput and the delay of backlogged packets. All found results show that our approach improves significantly the QoS of the system.**

Keywords: Game Theory, Markov Chain, Nash Equilibrium, Slotted ALOHA, ZigZag Decoding, Transmission Cost, Performance Evaluation

I. Introduction

Medium access control (MAC) mechanisms are designed for coordinating the access to common media and permission for transmission over them. These mechanisms are of significant importance since the wireless communication channel is very sensitive to collisions, and problems such as the hidden-terminal problem. The most popular multiple access protocols is probably the ALOHA family [1], Carrier Sense Multiple Access (CSMA) [2], and their corresponding variations [3, 4]. They have been widely studied as efficient methods to coordinate the medium access among competing users. To overcome contention issues between competitive nodes, various mechanisms have been implemented. For instance, to reduce contention under Slotted ALOHA (SA) mechanism, a user transmits a packet with certain probability during each time slot. However, in CSMA mechanism, a user maintains a back-off window and waits for a random amount of time bounded by the back-off window before a transmission (or retransmission).

Game theory has been widely recognized as an important tool for studying and analyzing the behavior of complicated and interacting systems [5, 6, 7, 8]. In wireless systems, nodes are usually not exactly aware of number of nodes in the network and each node can obtain some limited information about channel state (e.g. collided packets, busy or idle state of channel) through listening to channel. In such conditions the best thing a node can do, is to maximize its own payoff function. Therefore, for modeling such situation, non-cooperative game models [9] are the best choice.

The main motivation of this work is to provide a stochastic game framework to systematically study the behavior of selfish nodes in wireless networks. We propose herein a game theoretic approach of the SA mechanism combined with a technique called ZigZag Decoding (ZD) and a pricing mechanism. We start by analyzing the cooperative model of the proposed mechanism, where all users are assumed to optimize the same utility function. We assume that users maximize the global throughput of the system which is equivalent to minimize the delay of transmitted packets. Unfortunately, in this case, the delay of backlogged packets becomes huge when arrival probability is close to one, which is unfair utilisation of the system. We propose to solve this problem by minimizing the delay of backlogged packets but at the expense of sacrificing the global throughput. This tradeoff between the delay of backlogged packets and the global throughput is optimized using an adjustable utility function.

Next, we model the system using a non-cooperative model; we take as a utility function the average throughput of one tagged node. Then, we investigate the impact of adding a transmission cost (which could be expressed in terms of the energy consumption). Firstly, fixed Transmission Costs (TCs) are included in the model in order to mitigate the selfish behavior of competing nodes. Secondly, we optimize the TC to achieve the same performances as the cooperative game. Finally, we propose an adjustable TC to support all kind of traffics (e.g. real-time and non-real-time traffic). We evaluate our results using Matlab and compare them to the basic SA mechanism with a transmission cost (SA-TC) [10]. We also compare our results with those of the cooperative and the non-cooperative Slotted ALOHA mechanism combined with ZigZag Decoding (SA-ZD) without transmission cost [11, 12].

The rest of the paper is organized as follows. In Sect. II, we discuss some recent empirical studies in the context of selfish behavior in wireless networks. We present a brief overview of the proposed mechanism in Sect. III. In Sect. IV we model the system using Markov chains and cooperative games, and in Sect. V, we formulate a non-cooperative model of the SA-ZD mechanism in which we integrate the concept of TC. Section VI is dedicated to numerical results where we investigate the impact of adding the TC on the performance metrics of the system. Finally, Sect. VII concludes the paper.

II. Related work

Considerable work has been done for analyzing the behavior of selfish nodes in wireless networks. Below we highlight some of this literature.
Altman et al. [10] proposed to add a TC to every transmission attempt, the authors show that in non-cooperative game this pricing can be used to get the same throughput given by cooperative game. The same idea as [10] have been proposed by Karouit et al. in [13], where they studied the Binary Exponential Backoff (BEB) mechanism with Multiple Power Levels (MPLs). The resulting mechanism named MPL-BEB shows that under heavy traffic the Mobile Stations (MSs) act selfishly by attempting to get access to the channel using a retransmission probability close to 1. Thus, introducing the TC allows to control such a behavior and improve the performance of the system. In [14], the authors studied the influence of TCs on the behavior of selfish nodes in wireless Local Area Networks (LANs). Unlike [10], they assumed that active nodes know the current number of backlogged packets in the system. Therefore, nodes can decide whether or not to accept the new arriving packet, depending on the expected cost of successful transmission.
Zaaloul et al. studied the SA-ZD mechanism as a cooperative game in [11] and as a non-cooperative game in [12]. Our studies extend [11] by introducing an adjustable objective function in order to optimize the the existing tradeoff between the global throughput and the delay of backlogged packets. Furthermore, we extend [12, 15] by introducing the TC into the SA-ZD mechanism in order to mitigate the selfish behavior of competing nodes and hence improve the Quality of Service (QoS) of the system.

III. Proposed Mechanism

The Slotted ALOHA protocol is probably one of the most popular in the multiple access protocols family. It has long been used as random distributed medium access for radio channels. Indeed, it is so simple that its implementation is straightforward, and many local area networks of today implement some variants of Slotted ALOHA. In these protocols, if two or more packets are sent simultaneously then they collide. Packets that are involved in a collision are backlogged and retransmitted later.
We propose to combine the Slotted ALOHA mechanism with a technique called ZigZag Decoding [16]. ZigZag requires no changes to the MAC layer and introduces no overheard in the case of no collision. If there is no collusion, ZigZag acts like a typical random access method. Furthermore, it achieves the same performance as if the colliding packets were a priori scheduled in separate time slots [16]. If exactly two nodes collide in a given slot, we assume that the next slot is reserved to the same nodes to collide again in order to trigger the ZigZag Decoding mechanism, so in the second slot of ZigZag all other stations remain silent. Thus, the resulting mechanism works as follows:

- Time is divided into slots of one packet duration.
- The frame size is either one or two slots.
- If one node attempts transmission during a slot, the transmission is successful.
- If two nodes attempt transmission during a slot, the transmission is successful by ZD.
- Otherwise, a collision occurs and packets involved in a collision are lost.
- Collided packets are retransmitted after a random delay.
- If a new packet arrives during a slot, it will be transmitted in the next slot.
- If a transmission has failed, node becomes backlogged.
- For simplicity, we first assume that transmissions are cost free, but later costs are introduced in order to reduce selfishness.

In the proposed mechanism, exactly one of the following four events happens (Idle, Success, ZigZag, Collision). Thus there are three immediate feedback states:

- 0: idle (i.e. no packets attempted transmission),
- 1: success (i.e. exactly one packet attempted transmission),
- ZigZag: exactly 2 packets attempted transmission,
- C: more than tow packets attempted transmission.

IV. Problem Formulation for a Cooperative Game

In this section, we describe a cooperative Slotted ALOHA combined with a ZigZag Decoding technique and we construct a Markov model based on [17], from which performance parameters are measured.
We consider a cellular system where M bufferless nodes transmit over a common channel to a base station. It is assumed that time is divided into fixed length slots, and transmission of one packet takes a single slot. The arrival flow of packets to each source follows a Bernoulli process with parameter p_a (i.e. at each time slot, there is a probability p_a of a new arrival at a source, and all arrivals are independent), for simplicity purpose we restrict to the case where p_a is the same for all nodes.
Let us denote by $Q_a(i,N)$ the probability that i unbacklogged nodes transmit packets in a given slot.

$$Q_a(i,N) = \binom{M-N}{i} (1-p_a)^{(M-N-i)}\, p_a^i\,, \quad (1)$$

and let $Q_r(i, N)$ be the probability that i out of backlogged nodes retransmit packets in a given slot.

$$Q_r(i, N) = \binom{N}{i} (1 - q_r)^{(N-i)}\, q_r^i \,. \qquad (2)$$

We denote by N the number of backlogged nodes (or equivalently, of backlogged packets) at the beginning of a slot. For any choice of values $q_r \in]0, 1]$, the state process is a Markov chain for which the state space is $E = \{0, 1, ..., M\}$. Indeed, it is easy to check that the past and future are conditionally independent, given the present state. The transition diagram of the Markov Chain is given by Fig 1, where transition probabilities are given in appendix A.

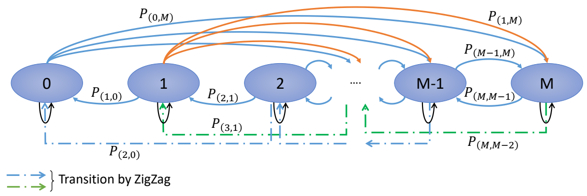


Figure. 1: Markov chain for the cooperative game problem

Since the state space is finite and all states communicate with each other, then the Markovian chain is ergodic, and therefore the stationary distribution exists. Let $\pi(p_a, q_r)$ be the corresponding vector of steady state probabilities where its N^{th} entry $\pi_N(p_a, q_r)$ denotes the probability of N backlogged nodes.
The steady state of the Markovian process is given by the following system

$$\begin{cases} \pi(p_a, q_r) = \pi(p_a, q_r) P(p_a, q_r), & \\ \pi_N(p_a, q_r) \geq 0, & N = 0, ..., M, \\ \displaystyle\sum_{N=0}^{M} \pi_N(p_a, q_r) = 1. & \end{cases} \qquad (3)$$

Using a simple iterative method, we can compute the stationary distribution from the system (3).

A. Frame Size

In SA-ZD mechanism, the frame size is either one or two slots. In the case of ZigZag the frame size is two slots otherwise it's one slot.

$$T = 2P_{ZigZag} + (1 - P_{ZigZag}) \,, \qquad (4)$$

where P_{ZigZag} is the probability of the event ZigZag, it is defined by

$$\begin{aligned} P_{ZigZag} = \sum_{N=0}^{M} & [Q_a(0, N) Q_r(2, N) + Q_a(2, N) Q_r(0, N) \\ & + Q_a(1, N) Q_r(1, N)]\, \pi_N(p_a, q_r) \,. \end{aligned} \qquad (5)$$

B. Optimization Problem for the Cooperative Game

In a cooperative game, nodes seek to optimize the same utility function (denoted by *objective* (p_a, q_r)). We propose herein four approaches to optimize the performances of the system. First, we assume that all nodes maximize the global throughput IV-B.1. Then, we take as objective function the delay of transmitted packets IV-B.2. Next, we consider that nodes minimize the delay of backlogged packets IV-B.3. Finally, we propose to compromise between the throughput and the delay of backlogged packets IV-B.4.

$$\max_{q_r \in [\epsilon, 1]} \textit{objective}\,(p_a, q_r) \text{ subject to:}$$

$$\begin{cases} \pi(p_a, q_r) = \pi(p_a, q_r).P(p_a, q_r), \\ \pi_N(p_a, q_r) \geq 0, \; N = 0, ..., M, \\ \displaystyle\sum_{N=0}^{M} \pi_N(p_a, q_r) = 1. \end{cases}$$

Singularity at $q_r = 0$: The only point where the Markov chain does not have a single stationary distribution is at $q_r = 0$, where it has three absorbing states: $N = M$, $N = M - 1$ and $N = M - 2$. All remaining states are transient (for any $p_a > 0$), and the probability to end at one of the absorbing states depend on the initial distribution of the Markov chain. We note that if the state $M - 1$ is reached then the throughput is p_a, otherwise if the state M is reached then the throughput equals 0, which means that it is a deadlock state. For $p_a > 0$ and $q_r = 0$, the deadlock state is reached with positive probability from any initial state other than absorbing states $M - 1$ and $M - 2$. We shall therefore exclude the case of $q_r = 0$ and optimize only on the range $\epsilon \leq q \leq 1$. We chose throughout this paper $\epsilon = 10^{-4}$.

Existence of a solution: The steady state probabilities $\pi(p_a, q_r)$ are continuous over $0 < q_r \leq 1$ which is not a close interval, therefore a solution may not exist. However, as we restrict to the closed interval $[\epsilon, 1]$ where $\epsilon > 0$, an optimal solution indeed exists. Therefore for any $\gamma > 0$, there exists some $q_r^* > 0$ which is γ-optimal. ($q_r^* > 0$ is said to be γ-optimal if it satisfies *objective*$(p_a, q_r^*) \geq$ *objective*$(p_a, q_r^*) - \gamma$ for all $q_r \in [\epsilon, 1]$.)

1) Maximization of the Global Throughput

The throughput of the system is defined as the sample average number of packets that are successfully transmitted; it is given almost surely by the constant

$$\begin{aligned} Th(p_a, q_r) =& \frac{1}{T} \sum_{N=1}^{M} [P_{succ}^N \pi_N(p_a, q_r) \qquad (6) \\ & + (Q_a(1, 0) + 2Q_a(2, 0))\, \pi_0(p_a, q_r)] \\ =& \frac{p_a}{T} \sum_{N=0}^{M} (M - N) \pi_N(p_a, q_r) \,, \qquad (7) \end{aligned}$$

where

$$\begin{aligned} P_{succ}^N =& Q_a(0, N) Q_r(1, N) + Q_a(1, N) Q_r(0, N) \\ & + 2Q_a(0, N) Q_r(2, N) + 2Q_a(2, N) Q_r(0, N) \\ & + 2Q_a(1, N) Q_r(1, N) \,. \end{aligned}$$

Note that the equation (6) represent the expected number of arrivals at a time T and the equation (7) represent the expected number of departures.

As in [10], the maximization of the global throughput leads to a huge delay of backlogged packets. Indeed, when we maximize the global throughput (in the case of heavy load), the system give more priority to new arrival packets.

2) *Minimization of the Delay of Transmitted Packets*

We can define the delay as the average time, in slots, that a packet takes from its source to the receiver. By Little's formula [18], the delay is given by

$$D(p_a, q_r) = \frac{Th(p_a, q_r) + S_B(p_a, q_r)}{Th(p_a, q_r)} = 1 + \frac{S_B(p_a, q_r)}{Th(p_a, q_r)} \,, \tag{8}$$

where $S_B(p_a, q_r) = \sum_{N=0}^{M} N\pi_N(p_a, q_r)$ is the average number of backlogged packets. The analysis of the equations (6) and (8) shows that maximizing the throughput is equivalent to minimizing the average delay of transmitted packets.

3) *Minimization of the Delay of Backlogged Packets*

An interesting alternative for measuring the performance of the system is to analyze the ability to serve packets awaiting retransmission. It has a great interest especially for real-time applications. Let $Th_{succ}(p_a, q_r)$ be the average throughput of new packets arrived (crowned with success), so the average throughput for backlogged packets is given by

$$Th_B(p_a, q_r) = Th(p_a, q_r) - Th_{succ}(p_a, q_r) \,, \tag{9}$$

where $Th_{succ}(p_a, q_r)$ is defined by

$$\begin{aligned} Th_{succ}(p_a, q_r) = & \frac{1}{T} \sum_{N=0}^{M-1} [Q_a(1, N)Q_r(0, N) \\ & + Q_a(1, N)Q_r(1, N) \\ & + 2Q_a(2, N)Q_r(0, N)] \, \pi_N(p_a, q_r) \,. \end{aligned} \tag{10}$$

Thereafter, we can calculate the expected delay $D_B(p_a, q_r)$ of backlogged packets by applying Little's formula [18]. It is given by

$$D_B(p_a, q_r) = \frac{Th_B(p_a, q_r) + S_B(p_a, q_r)}{Th_B(p_a, q_r)} = 1 + \frac{S_B(p_a, q_r)}{Th_B(p_a, q_r)} \,. \tag{11}$$

4) *Proposed Objective Function*

Our results in [15] show that, in the case of non-cooperative game, a tradeoff exists between the throughput of the system and the delay of backlogged packets. Since the same tradeoff exist in the case of cooperative game, we propose an adjustable objective function with a parameter α in order to compromise between the throughput and the delay of backlogged packets.

$$objective_\alpha(p_a, q_r) = (1-\alpha)Th(p_a, q_r) + \alpha \frac{1}{D_B(p_a, q_r)} \,, \tag{12}$$

where $\alpha \in [0, 1]$. We note that $\alpha = 0$ (resp $\alpha = 1$) is equivalent to the case IV-B.1 (resp IV-B.3). Hence, for traffic requesting minimum delay, we shall choose $\alpha \to 1$, and for traffic requesting maximum throughput, we choose $\alpha \to 0$.

C. *Fairness Analysis of SA and SA-ZD*

As we mentioned before in the scenario IV-B.1, when the arrival probability is close to one ($p_a \to 1$) the system (in both mechanisms SA and SA-ZD) prioritizes the new arriving packets in order to maximize the throughput of the system. As a result, backlogged nodes stay backlogged for very long time (since $q_r^* = \epsilon$). To ensure fairness between nodes we propose to choose the same probability for backlogged and new arriving packets ($p_a \approx q_r^*$). In Fig. 2 we present the arrival probability (for SA and SA-ZD) which should be equal to the optimal retransmission probability for the team problem (in the scenario IV-B.1) as a function of number of nodes in the system. We observe that the arrival probability decreases when the number of nodes increases which means that all nodes should lower their transmission and retransmission probability when the system becomes congested in order to make everyone satisfied.

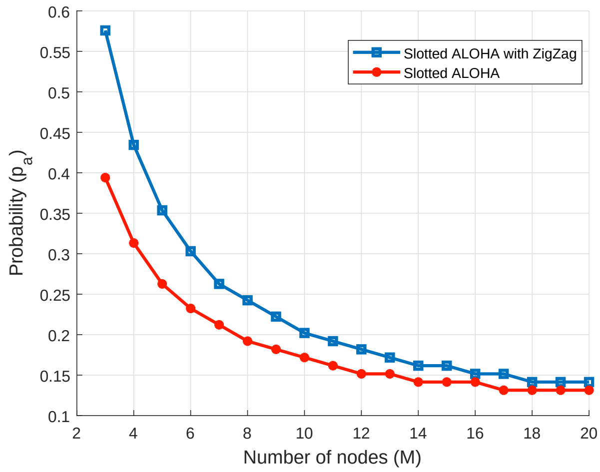


Figure. 2: Arrival probability p_a as a function of number of nodes

V. A Non-cooperative Game

In many cases Slotted ALOHA system is usually a decentralized entity, so the cooperative model is not efficient any more. We will develop a model for decentralized non cooperative game which is more powerful and appropriate to Slotted ALOHA. The Nash equilibrium concept replaces the concept of optimality in the team problem.

A. *Problem Formulation for a Non-cooperative Game*

We take the same notations that we defined before for the team problem, and we consider $M + 1$ bufferless nodes that compete to get access to a shared channel. Let $\mathbf{q_r} = (q_r^1, q_r^2, ..., q_r^{M+1})$ be a vector of retransmission probabilities for all users. We defined by $([\mathbf{q_r}]^{-i}, q_r^i)$ a retransmission policy where user i retransmits at any slot with probability q_r^i and any other node j retransmits with probability q_r^j for all $j \neq i$. We assume that all the M nodes retransmit with a given probability $[\mathbf{q_r}]^{-(\mathbf{M+1})} = (q_r, q_r, ..., q_r)$, whereas the node $M + 1$ retransmits with probability $q_r^{(M+1)}$.

In a non-cooperative game, each player attempts to optimize its utility (denoted by $objective_i(\mathbf{q_r})$), by either maximizing its own throughput or minimizing the expected delay of its packets. Our objective is to find a symmetric equilibrium policy $\mathbf{q_r^*} = (q_r, q_r, ..., q_r)$ such that for any user i and any retransmission probability $q_r^i \neq q_r$

$$objective_i(\mathbf{q_r^*}) \geq objective_i(q_r, ..., q_r^i, ..., q_r) \, . \qquad (13)$$

Due to symmetry, verifying (13) for a single player is a sufficient condition for $\mathbf{q_r^*} = (q_r, q_r, ..., q_r)$ to be an equilibrium. Hereafter, we choose the player $M+1$ to be our tagged user. We define the set of best response strategies of user $M+1$ by

$$Q^{M+1}(\mathbf{q_r}) = \underset{q_r^{(M+1)} \in [\epsilon, 1]}{\operatorname{argmax}} \left\{ objective_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right) \right\} \, , \quad (14)$$

where $\bar{\mathbf{q}}_{(M+1)} = \left([\mathbf{q_r}]^{-(M+1)}, q_r^{(M+1)} \right)$ denote the retransmission policy, and the maximization is taken with respect to $q_r^{(M+1)}$. Then $\mathbf{q_r^*}$ is a symmetric equilibrium if

$$\mathbf{q_r^*} \in Q^{M+1}(\mathbf{q_r^*}) \, . \qquad (15)$$

To compute the performance metrics we shall use a bi-dimensional Markov chain improved by ZigZag Decoding. The transition probability diagram is depicted in Fig. 3 and transition probabilities are given in appendix B. Let the first state component be the number of backlogged packets among the users $1, ..., M$, and the second component be the number of backlogged packets of the user $M+1$ (either 1 or 0). For any choice of values $q \in]0, 1]$, the state process is a Markov chain that contains a single ergodic sub-chain (and possibly transient states as well). Indeed, it is easy to check that the past and future are conditionally independent, given the present state (Markov property).

Since the state space is finite and all states communicate with each other, then the Markovian chain is ergodic, and therefore the stationary distribution exists. Let $\pi\left(\bar{\mathbf{q}}_{(M+1)}\right)$ be the corresponding vector of steady state probabilities where its N^{th} entry $\pi_{N,a}\left(\bar{\mathbf{q}}_{(M+1)}\right)$ denotes the probability that the state of the system is (N, a). Then the steady state of the Markovian process is given by the following system

$$\begin{cases} \pi\left(\bar{\mathbf{q}}_{(M+1)}\right) = \pi\left(\bar{\mathbf{q}}_{(M+1)}\right) P\left(\bar{\mathbf{q}}_{(M+1)}\right), \\ \pi_{N,a}\left(\bar{\mathbf{q}}_{(M+1)}\right) \geq 0, \qquad N = 0, ..., M \text{ and } a = 0, 1 \\ \sum_{N=0}^{M} \sum_{a=0}^{1} \pi_{N,a}\left(\bar{\mathbf{q}}_{(M+1)}\right) = 1. \end{cases} \quad (16)$$

B. Frame Size

The frame size in a non-cooperative SA-ZD is either one or two slots and it is defined by

$$T' = 2P'_{ZigZag} + (1 - P'_{ZigZag}) \, , \qquad (17)$$

where P'_{ZigZag} is the probability that two packets are transmitted by ZD, it is given by

$$P'_{ZigZag} = \sum_{N=0}^{M} P_1 \pi_{N,0}\left(\bar{\mathbf{q}}_{(M+1)}\right) + P_2 \pi_{N,1}\left(\bar{\mathbf{q}}_{(M+1)}\right) \, , \qquad (18)$$

where

$$\begin{aligned} P_1 =& p_a \left[Q_a(1,N) Q_r(0,N) + Q_a(0,N) Q_r(1,N) \right] \\ &+ (1 - p_a) \left[Q_a(0,N) Q_r(2,N) + Q_a(2,N) Q_r(0,N) \right. \\ &\left. + Q_a(1,N) Q_r(1,N) \right], \end{aligned}$$

and

$$\begin{aligned} P_2 =& q_r^{(M+1)} \left[Q_a(1,N) Q_r(0,N) + Q_a(0,N) Q_r(1,N) \right] \\ &+ (1 - q_r^{(M+1)}) \left[Q_a(0,N) Q_r(2,N) \right. \\ &\left. + Q_a(2,N) Q_r(0,N) + Q_a(1,N) Q_r(1,N) \right]. \end{aligned}$$

C. Performance Metrics for Non-cooperative Game Problem

We can now calculate the number of backlogged packets of our tagged user $M+1$ as follows

$$S_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right) = \sum_{N=0}^{M} \pi_{N,1}\left(\bar{\mathbf{q}}_{(M+1)}\right) \, . \qquad (19)$$

The average throughput of user $M+1$ is given by

$$THp_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right) = \frac{p_a}{T'} \sum_{N=0}^{M} \pi_{N,0}\left(\bar{\mathbf{q}}_{(M+1)}\right) \, . \qquad (20)$$

By Little's formula [18], the delay is given by

$$D_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right) = 1 + \frac{S_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right)}{THp_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right)} \, . \qquad (21)$$

The average throughput of backlogged packets of user $M+1$ is

$$\begin{aligned} &THp^B_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right) \\ &= \frac{1}{T'} \sum_{N=0}^{M} \sum_{N'=0}^{M} P_{(N,0)(N',1)}\left(\bar{\mathbf{q}}_{(M+1)}\right) \pi_{N,0}\left(\bar{\mathbf{q}}_{(M+1)}\right) \, . \end{aligned} \quad (22)$$

The delay of backlogged packets of user $M+1$ is given by

$$D^B_{(M+1)} = 1 + \frac{S_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right)}{THp^B_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right)} \, . \qquad (23)$$

D. Transmission Cost

In [13] and [12], we observe that as the arrival probability increases, the users tend to be more aggressive at equilibrium. This results in a dramatic decreases in the system's throughput. This is mainly due to the fact that the users act selfishly by attempting to access the channel with a retransmission probability q_r close to 1, which yields more energy consumption and more collisions. To avoid this collapse network, it is required to control the behavior of users. The main idea behind is to reduce the failure probability by limiting the aggressiveness of the competing nodes. Towards this end, we propose to associate a TC denoted by C (which can, in particular, represent the battery power cost) to each transmission and retransmission attempt.

For illustrative purpose, we consider the example of one player J_1 versus a couple of players J_2 and J_3 together, as

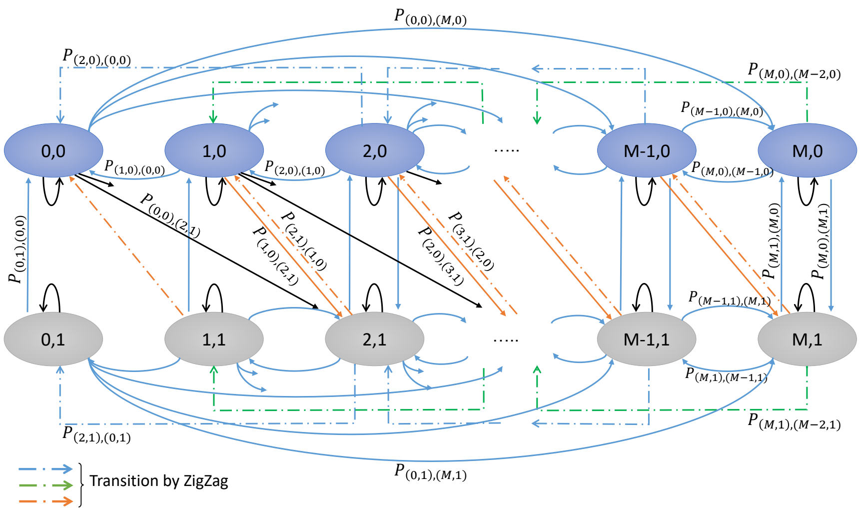


Figure. 3: Bi-dimensional Markov chain for the non-cooperative game problem

shown in Table 1. The first player can undertake two actions, either transmit or wait, while players $\{J_2, J_3\}$ can undertake three actions, either they both transmit (TT), they both wait (WW), or one transmits and the other waits (TW). If exactly one player decides to transmit while the others decide to wait, he receives $(1-C)$ and the others receive 0. If exactly two players decide to transmit, each one receives $1-C$ (due to ZD) while the remaining player (the one who decides to wait) receives 0. If all the three players decide to transmit, a collision occurs, and each one of them receives $-C$.

Table 1: Three nodes random access game with transmission cost

player J_1	players $\{J_2, J_3\}$		
	TT	TW	WW
Transmit (T)	(-C,-2C)	(1-C,1-C)	(1-C,0)
Wait (W)	(0,2(1-C))	(0,1-C)	(0,0)

At the steady state, when the node $M+1$ transmits successfully, it gains $(1-C)THp_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right)$. Similarly, when the transmission fails, it pays a cost $Cq_r^{M+1}\sum_{N=0}^{M}\pi_{N,1}\left(\bar{\mathbf{q}}_{(M+1)}\right)$. Thus, the average utility of node $M+1$ is given by

$$\begin{aligned} objective_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right) =& (1-C)THp_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right) \\ & - Cq_r^{M+1}\sum_{N=0}^{M}\pi_{N,1}\left(\bar{\mathbf{q}}_{(M+1)}\right). \end{aligned} \tag{24}$$

Therefore, in order to maximize its own profit, node (M+1) is now faced with the following challenge

$$maximize_{q_r^{(M+1)}\in[\epsilon,1]}\left\{objective_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right)\right\}. \tag{25}$$

We define as we did before, the set of best response strategies of user $M+1$

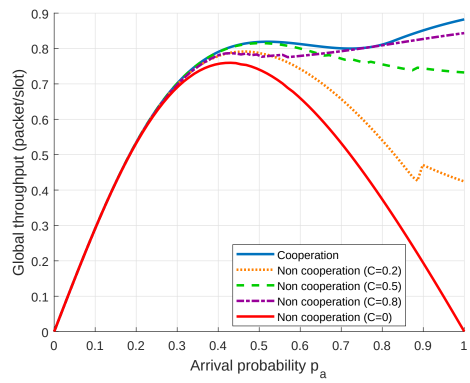
$$\mathcal{Q}^{M+1}\left(\mathbf{q_r}\right) = \underset{q_r^{(M+1)}\in[\epsilon,1]}{\operatorname{argmax}}\left\{objective_{(M+1)}\left(\bar{\mathbf{q}}_{(M+1)}\right)\right\}, \tag{26}$$

then we seek the value $\mathbf{q_r^*}$ of retransmission probability that satisfies

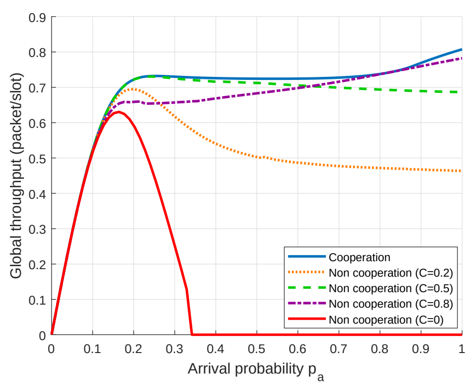
$$\mathbf{q_r^*} \in \mathcal{Q}^{M+1}\left(\mathbf{q_r^*}\right), \tag{27}$$

which is the Nash equilibrium for the non-cooperative game.

VI. Numerical Results

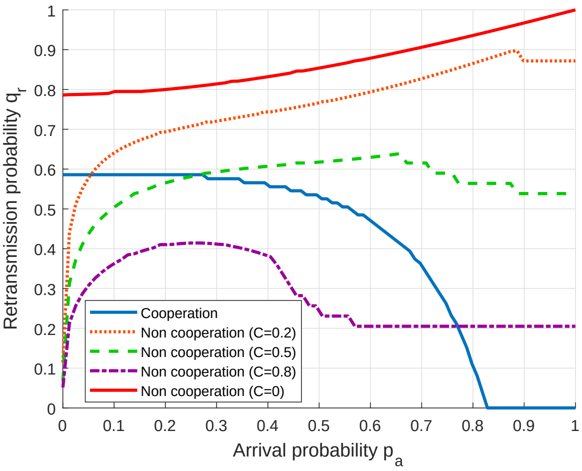


(a) for $M+1=3$

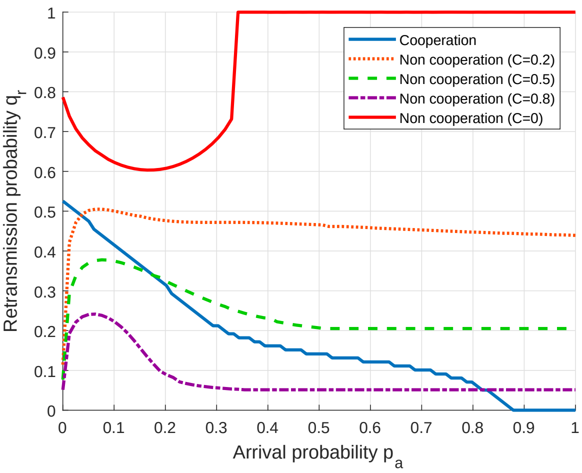


(b) for $M+1=6$

Figure. 4: Global throughput versus arrival probability for 3 (a) and 6 (b) nodes, under different values of $C = [0\ 0.2\ 0.5\ 0.8]$

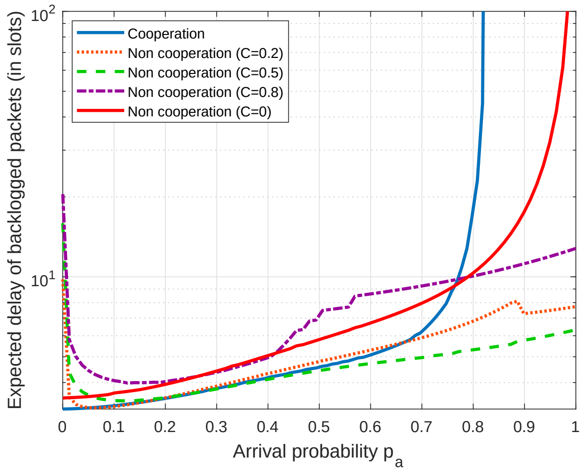


(a) for $M + 1 = 3$

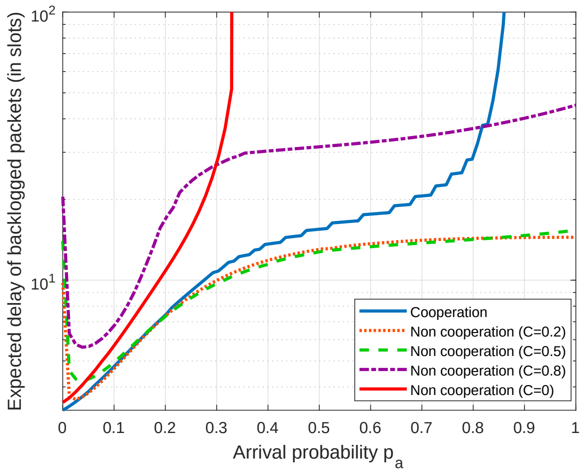


(b) for $M + 1 = 6$

Figure. 5: The retransmission probabilities as a function of arrival probability for 3 (a) and 6 (b) nodes, under different values of $C = [0\ 0.2\ 0.5\ 0.8]$



(a) for $M + 1 = 3$



(b) for $M + 1 = 6$

Figure. 6: Expected delay of backlogged packets as a function of the arrival probability p_a for 3 (a) and 6 (b) nodes, under different values of $C = [0\ 0.2\ 0.5\ 0.8]$

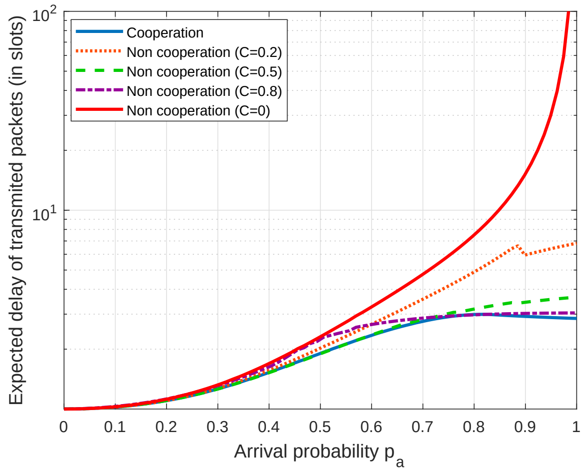
A. *Fixed Transmission Cost*

In this section we compare the performance metrics of the Slotted ALOHA mechanism combined with ZigZag Decoding, in one hand as a cooperative game problem, and in the other hand as a non-cooperative game problem with various transmission costs ($C = 0,\ 0.2,\ 0.5,\ 0.8$). We show how this transmission cost can affect performance metrics of the system.

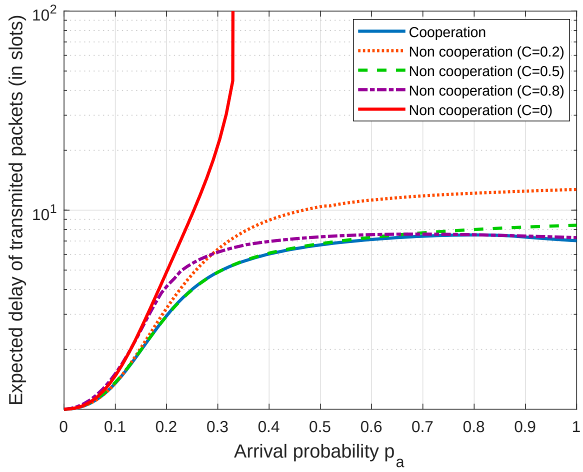
We depict in Fig. 4 and Fig. 5 the global throughput at Nash equilibrium and Nash equilibrium retransmission probability, respectively, as a function of p_a for different values of C. We compare the non-cooperative case (in which various transmission costs have been added $C = 0,\ 0.2,\ 0.5,\ 0.8$) with the cooperative case. We take the cooperative game (section IV-B.1) as a benchmark to measure the impact of adding a transmission cost on the selfish behavior of competing nodes. As shown in Fig. 4 the Slotted ALOHA combined with ZigZag Decoding and transmission cost proves very effective, especially for large arrival probability p_a. In Fig. 5 we see a decrease on the equilibrium retransmission probability as the cost increases which means that the pricing mechanism strongly affect the behavior of nodes.

We plot in Fig. 6 and Fig. 7, the expected delay of backlogged and successfully transmitted packets, respectively, as a function of arrival probability p_a for different values of C. We note that transmission cost leads to a bounded delay. However, a large pricing could have a negative impact (i.e. huge delay) since nodes will never transmit when the transmission cost is greater or equals to the gain obtained in a successful transmission. Furthermore, we may wonder here (when p_a is close to 1, see Fig. 6) why the pricing mechanism could looks very effective than cooperative game, this is mainly due to the fact that, the cooperative system prioritizes the new arrival packets (since p_a is great) in order to maximize the global throughput. This priority mechanism does not appear in the non-cooperative game, therefore, introducing a transmission cost in the non-cooperative game makes the system more effective.

For all above, we note that the equilibrium depends on the transmission cost C, that's why we should carefully choose the cost C which gives the best equilibrium.

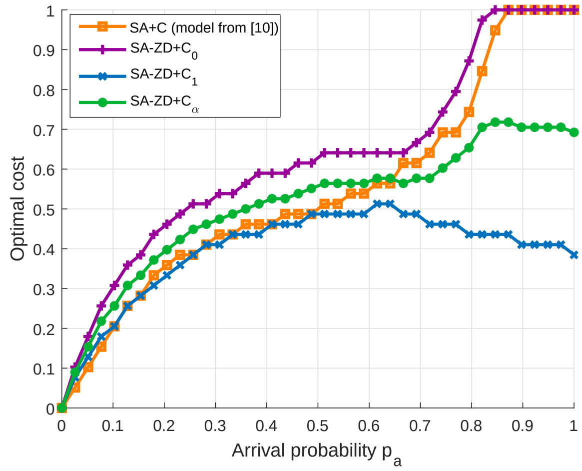


(a) for $M+1=3$

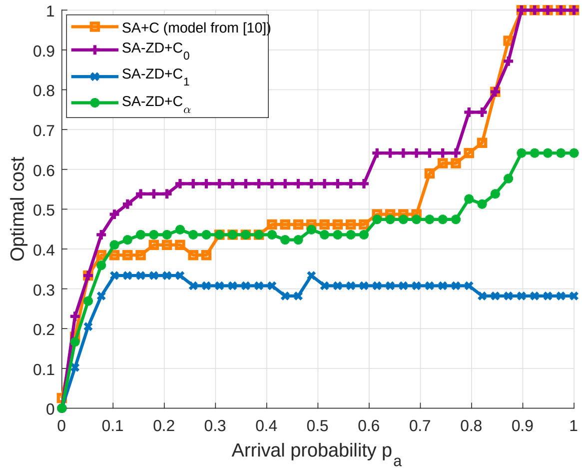


(b) for $M+1=6$

Figure. 7: Expected delay of packets that are successfully transmitted as a function of the arrival probability p_a for 3 (a) and 6 (b) nodes, under different values of $C = [0\ 0.2\ 0.5\ 0.8]$



(a) for $M+1=3$



(b) for $M+1=10$

Figure. 8: The optimal transmission cost as a function of arrival probability p_a for 3 (a) and 10 (b) nodes

B. Optimization on the Transmission Cost

According to the previous results, we observe that for different values of arrival probability p_a, we obtain different costs C which gives the best throughput. Therefore, we seek the cost C that is necessary for the equilibrium retransmission probabilities to coincide with those obtained in the team problem (Sect. IV). First we define C_0 (28) to be the cost correspond the best average throughput (which gives the same results as Sect. IV-B.1. Then, we define the cost C_1 (29) that minimizes the delay of backlogged packets (same performances as cooperative game Sect. IV-B.3) and finally C_α (30) is defined to be an adjustable cost with parameter α in order to compromise between the throughput and the delay of backlogged packets. The cost C_α generalizes the cost C_3 In [15]. Indeed, it takes all the values between C_0 and C_1. Therefore, in numerical results we focus on $\alpha = \frac{1}{2}$.

$$\max_{C\in[0,1]} \left\{THp_{(M+1)}\left(p_a, q_r^*(C)\right)\right\} , \tag{28}$$

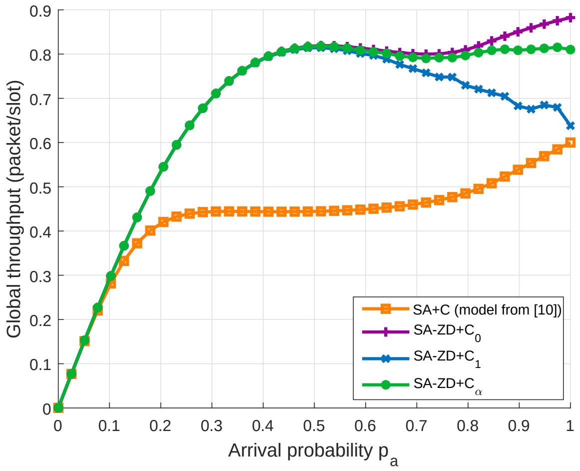
$$\max_{C\in[0,1]} \left\{\frac{1}{D^B_{(M+1)}\left(p_a, q_r^*(C)\right)}\right\} , \tag{29}$$

where $q_r^*(C)$ is the Nash equilibrium which depends on the transmission cost.

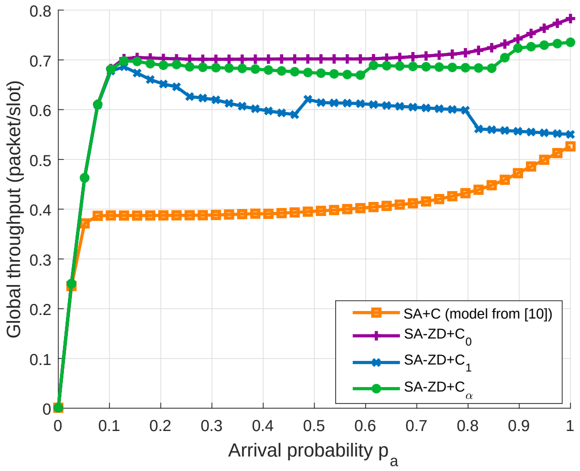
$$C_\alpha = (1-\alpha)C_0 + \alpha C_1 . \tag{30}$$

We depict in Fig. 8a and Fig. 8b, the optimal transmission cost C_0, C_1 and $C_{\frac{1}{2}}$, as a function of arrival probability, respectively, for 3 and 10 nodes. In both cases, we note that C_0 reaches the congestion cost (i.e. $C = 1$) when the arrival probability p_a is too large, In fact, the system prioritizes the new arriving packets in order to maximize the global throughput. Whereas C_1 and $C_{\frac{1}{2}}$ never reach 1 so that the backlogged packets are just as privileged as the new arriving packets.

We depict in Fig. 9 and Fig. 10, respectively, the aggregate throughput and the retransmission probability, as a function of arrival probability, for 3 and 10 nodes. We see that using the cost C_0 we achieve the maximum global throughput.

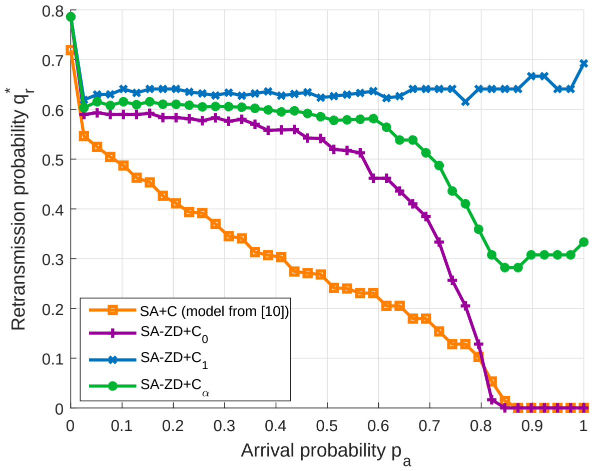


(a) for $M + 1 = 3$

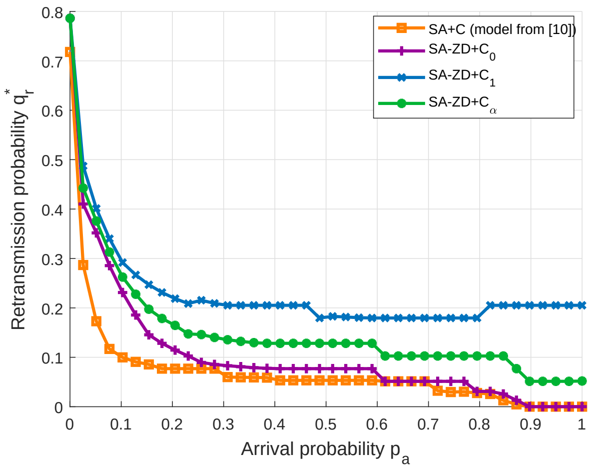


(b) for $M + 1 = 10$

Figure. 9: The global throughput in the game case with various optimal transmission cost



(a) for $M + 1 = 3$



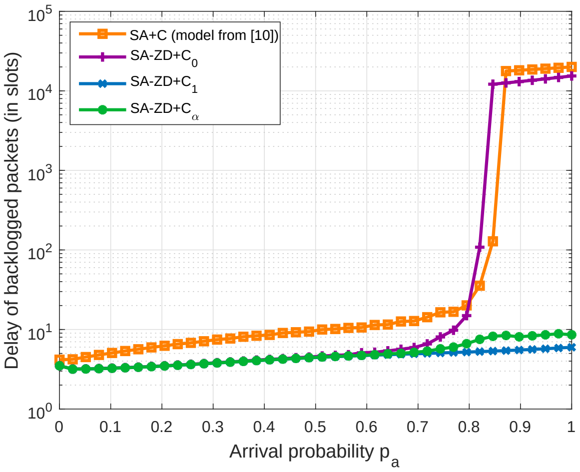
(b) for $M + 1 = 10$

Figure. 10: The retransmission probability in the game case with various optimal transmission cost

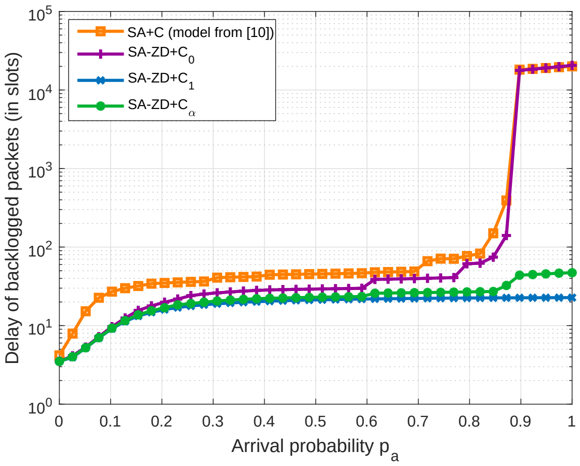
However, when we use the cost C_1 we obtain a minimum delay of backlogged packets, and finally when the cost $C_{\frac{1}{2}}$ is used a compromise is achieved, in return, this compromise is compensated with a bounded delay of backlogged packets (see Fig. 11). In Fig. 10, we see a visible and significant improvement of the retransmission probability and as a consequence a decrease in selfish behavior of competing nodes. In Fig. 11, we plot the delay of backlogged packets as a function of arrival probability. As we discussed before, we see that the delay of backlogged packets when using C_0 is huge even if the corresponding throughput is maximal. On the other side, the Fig. 12 shows the expected delay of transmitted packets and confirms that the system prioritizes the new arriving packets when using C_0. However, for the case of C_1 and $C_{\frac{1}{2}}$ there is no priority mechanism.

VII. Conclusion

In this paper, we have provided a mathematical model based on game theory and Markov chains in order to analyze the impact of transmission cost in SA-ZD mechanism. In the first part, we studied the system as a cooperative game where nodes are considered to optimize the same utility function. Four approach have been analyzed herein to optimize the performances of the system. We showed numerically that a tradeoff exists between the global throughput of the system and the delay of backlogged packets in SA and SA-ZD mechanisms. Next, we proposed an adjustable objective function in order to optimize this tradeoff. Then, we have proposed to control the arrival probability in order to ensure fairness in the system. In the second part, we have constructed a non-cooperative model using a bi-dimensional Markov chain improved by ZD, where each node is seeking to optimize its own utility function. Our numerical results showed that the non-cooperative system is not efficient since the overall network performance decreases. Therefore, we proposed to introduce fixed TCs in the non-cooperative model in order to mitigate the selfish behavior of competing nodes. Then, we optimized the TC to achieve the same performances as the cooperative game. Finally, we proposed to adjust the optimal TC to support various traffic requirements.

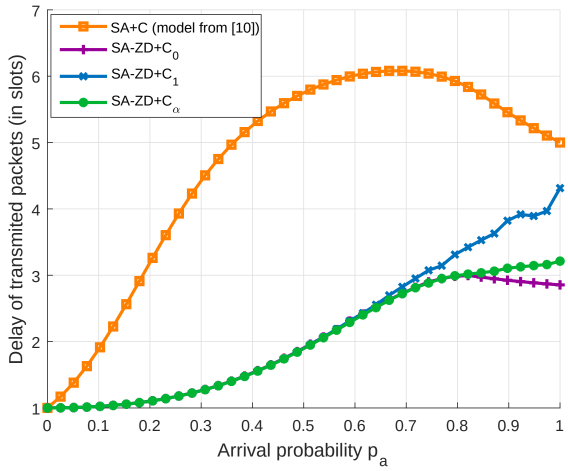


(a) for $M+1=3$

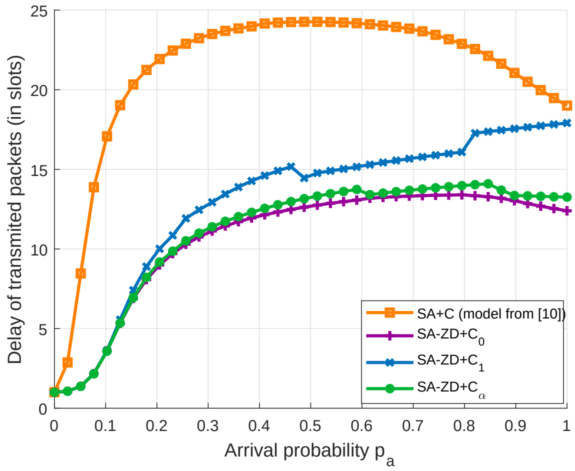


(b) for $M+1=10$

Figure. 11: Expected delay of backlogged packets in the game case with various optimal transmission cost



(a) for $M+1=3$



(b) for $M+1=10$

Figure. 12: Expected delay of packets that are successfully transmitted in the game case with various optimal transmission cost

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Appendix A: Transition Probabilities for Cooperative Game Problem

$$P_{(N,N+i)} = \begin{cases} Q_a(i,N), \quad 3 \leq i \leq M-N, \\ Q_a(1,N)\left(1 - Q_r(0,N) - Q_r(1,N)\right), \quad i=1, \quad 2 \leq N \leq M-1, \\ Q_a(2,N)\left(1 - Q_r(0,N)\right), \quad i=2, \quad 1 \leq N \leq M-2, \\ Q_a(0,N)\left[1 - Q_r(1,N) - Q_r(2,N)\right] + Q_a(1,N)Q_r(0,N) + Q_a(2,N)Q_r(0,N), \quad i=0, \\ Q_a(0,N)Q_r(1,N) + Q_a(1,N)Q_r(1,N) \quad i=-1, \quad 1 \leq N \leq M, \\ Q_a(0,N)Q_r(2,N), \quad i=-2, \quad 2 \leq N \leq M, \\ 0, \quad \text{otherwise.} \end{cases}$$

Appendix B: Transition Probabilities for Non-cooperative Game Problem

$$P_{(N,a)(N+i,b)} = \begin{cases} \left.\begin{array}{ll} Q_a(i,N), & a=b=1, \\ Q_a(i,N)(1-p_a), & a=b=0, \\ Q_a(i,N)p_a, & a=0, b=1, \end{array}\right\} \text{if } 3 \leq i \leq M-N, \\ \left.\begin{array}{ll} Q_a(1,N)\left[1 - Q_r(0,N) - Q_r(1,N)\right] + q_r^{M+1}Q_a(1,N)Q_r(1,N), & a=b=1, \\ (1-p_a)Q_a(1,N)\left[1 - Q_r(0,N) - Q_r(1,N)\right], & a=b=0, \\ p_aQ_a(1,N)\left[1 - Q_r(0,N)\right], & a=0, b=1, \end{array}\right\} \text{if } i=1, \\ \left.\begin{array}{ll} Q_a(2,N)\left[1 - Q_r(0,N)\right] + q_r^{M+1}Q_a(2,N)Q_r(0,N), & a=b=1, \\ (1-p_a)Q_a(2,N)\left[1 - Q_r(0,N)\right], & a=b=0, \\ p_aQ_a(2,N), & a=0, b=1, \end{array}\right\} \text{if } i=2, \\ \left.\begin{array}{ll} (1-q_r^{M+1})Z + q_r^{M+1}Q_a(0,N)\left[1 - Q_r(0,N) - Q_r(1,N)\right], & a=b=1, \\ (1-p_a)Z + p_a\left[Q_a(0,N) + Q_a(1,N)\right]Q_r(0,N), & a=b=0, \\ p_aQ_a(0,N)\left[1 - Q_r(0,N) - Q_r(1,N)\right], & a=0, b=1, \\ q_r^{M+1}\left[Q_a(0,N) + Q_a(1,N)\right]Q_r(0,N), & a=1, b=0, \end{array}\right\} \text{if } i=0, \\ \left.\begin{array}{ll} (1-q_r^{M+1})Q_r(1,N)\left[Q_a(0,N) + Q_a(1,N)\right], & a=b=1, \\ (1-p_a)Q_r(1,N)\left[Q_a(0,N) + Q_a(1,N)\right] + p_aQ_r(1,N)Q_a(0,N), & a=b=0, \\ q_r^{M+1}Q_r(1,N)Q_a(0,N), & a=1, b=0, \end{array}\right\} \text{if } i=-1, \\ \left.\begin{array}{ll} (1-q_r^{M+1})Q_r(2,N)Q_a(0,N), & a=b=1, \\ (1-p_a)Q_r(2,N)Q_a(0,N), & a=b=0, \end{array}\right\} \text{if } i=-2, \\ 0, \quad \text{otherwise,} \end{cases}$$

where: $Z = Q_a(0,N)\left[1 - Q_r(1,N) - Q_r(2,N)\right] + Q_a(1,N)Q_r(0,N) + Q_a(2,N)Q_r(0,N)$.